Proceedings of the 36th Chinese Control Conference
July 26-28, 2017, Dalian, China

Asymptotically Stable Observer for Two-dimensional Systems with Multiple-Channel Faults

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Abstract: This paper proposes a novel observer design method for two dimensional systems with multiple channel faults. The two-dimensional systems are described by Fornasini-Marchesini local state-space second model (FM-II) and the faults exist in both state equation and measurement equation. This study considers two fault situations, according to whether the faults have the same source. By transforming the fault in measurement equation as augmented state, the FM-II model with faults can be rewritten into singular systems. Hence, two observers are proposed for the singular systems, and then the estimation of the faults can be obtained. The necessary and sufficient conditions for the existence of asymptotically stable observer are derived. Numerical examples are given to demonstrate the effectiveness of the proposed method.

Key Words: two dimensional systems, asymptotically stable observer, fault estimation, singular systems.

1 Introduction

Two dimensional (2-D) systems theory have profound engineering background and can be used to solve many complex problems, such as the achievement of multi-variable network [1], multidimensional digital filtering and signal processing [2], image data processing [3] and batch process control [4]. For 1-D systems, the problem of fault diagnosis has attracted significant attention with numerous published results [5-12]. However, the fault diagnosis problem on 2-D systems is far from being well-investigated. In practice, 2-D systems face tremendous threats of fault in engineering practice and have the needs of reliability and security, the 2-D fault diagnosis problem has been paid more and more attention.

As everyone knows, observer/filter-based methods play significant role in model-based fault detection and they can provide more information such as fault size and types.Therefore, there are important industrial and theoretical applications in fault diagnosis and fault tolerant control. Some valuable results have been obtained about this topic. For example, Using 2-D singular systems theory, the state and the fault in measurement equation of FMM-II model are estimated simultaneously in [13, 14]. In [15], for 2-D nonlinear systems with time-varying delays and system perturbations, the state and fault estimation are obtained simultaneously and used in fault reconstruction. In [16], based on generalized H_2 index, the results of fault detection for 2-D Markov jump systems are obtained under the condition of random packet dropout.

Motivated by the aforementioned discussion, this paper considers the problem of estimating the state and faults simultaneously for 2-D systems. Different with all the reported observers, new observers that include fault estimation in state equation are firstly presented. By using singular system approach and stability theory, we derive the necessary and sufficient conditions for the existence of asymptotically stable observers and calculate new observers gain matrices.

The main contributions of this paper can be summarized as follows. For 2-D systems that described by FM-II model, when faults in state equation and measurement equation occur simultaneously, a asymptotically stable new observer is first designed such that the faults can be estimated at the same time. Furthermore, the asymptotically stable observers are derived for two fault situations.

The rest of paper is organized as follows. In part 2, for considered systems, some definitions and lemmas are presented. Part 3 gives necessary and sufficient conditions of asymptotically stable observers for two fault situations. In part 4, the effectiveness of proposed method is validated by numerical examples. Part 5 summarizes the results of this study.

2 Problem formulation and preliminaries

Consider a 2-D system with faults described by the FM-II model:

$$\begin{cases} x(i+1,j+1) = A_1x(i,j+1) + A_2x(i+1,j) \\ +B_1u(i,j+1) + B_2u(i+1,j) \\ +M_1f(i,j+1) + M_2f(i+1,j) \\ y(i,j) = Cx(i,j) + f_s(i,j) \end{cases} \tag{1}$$

with unknown boundary conditions

$$\begin{aligned} &\sup\|x(i,0)\| < \infty, i = 0,1,\ldots \\ &\sup\|x(0,j)\| < \infty, j = 1,2,\ldots \end{aligned} \tag{2}$$

where $x(i,j)\in R^n$, $y(i,j)\in R^p$, $u(i,j)\in R^m$ are the system state vector, measurement output vector, and input vector; $f(i,j)\in R^q$, $f_s(i,j)\in R^p$ are faults in state

This work is partially supported by National Natural Science Foundation in China (61374099) and Research Fund for the Taishan Scholar Project of Shandong Province of China.

equation and measurement equation, $f(i,j)$ can contain actuator fault and process fault, $f_s(i,j)$ represents sensor fault; $C, A_k, B_k, M_k, (k=1,2)$ are system matrices with appropriate dimensions. The dynamic characteristics of $f(i,j) \in R^q$ can be given as follows:

$$f(i+1,j+1) = A_{d1} f(i,j+1) + A_{d2} f(i+1,j) \tag{3}$$

where A_{d1} and A_{d2} are known matrices with appropriate dimensions.

In this paper, our objective is to design an observer to estimate system state and two kinds of faults simultaneously.

Define $\bar{x}(i+1,j+1) = \begin{bmatrix} x^T(i+1,j+1) & f_s^T(i+1,j+1) \end{bmatrix}^T$, Consider the following observer:

$$\begin{cases} z(i+1,j+1) = F_1 z(i,j+1) + F_2 z(i+1,j) \\ +H_1 u(i,j+1) + H_2 u(i+1,j) + G_1 y(i,j+1) \\ +G_2 y(i+1,j) + R_1 \hat{f}(i,j+1) + R_2 \hat{f}(i+1,j) \\ \hat{f}(i+1,j+1) = A_{d1}\hat{f}(i,j+1) + A_{d2}\hat{f}(i+1,j) \\ -K_{d1}\bar{C}z(i,j+1) - K_{d2}\bar{C}z(i+1,j) \\ \hat{\bar{x}}(i,j) = z(i,j) + Ty(i,j) \end{cases} \tag{4}$$

with unknown boundary conditions

$$\begin{aligned} &\sup\|z(i,0)\| < \infty, i = 0,1,... \\ &\sup\|z(0,j)\| < \infty, j = 1,2,... \end{aligned} \tag{5}$$

where $z(i,j) \in R^{n+p}$ and $\hat{\bar{x}}(i,j)$ are the state and output of observer; $\hat{\bar{x}}(i,j)$ and $\hat{f}(i,j)$ are the estimation of $\bar{x}(i,j)$ and $f(i,j)$; $F_k, H_k, G_k, R_k, K_{dk}, (k=1,2)$, T are the matrices that need to be designed. Define $e(i,j) = \bar{x}(i,j) - \hat{\bar{x}}(i,j)$, $e_d(i,j) = f(i,j) - \hat{f}(i,j)$, then introduce the definition of the 2-D observers.

Definition 1 ([13]). The observer (4) is an asymptotically stable observer for system (1), if for any system boundary conditions satisfying (2), for any observer boundary conditions satisfying (5), and for any input sequence $u(i,j)$, $\lim_{i,j\to\infty} e(i,j) = \mathbf{0}$, $\lim_{i,j\to\infty} e_d(i,j) = \mathbf{0}$ holds.

Lemma 1 ([17]). Define the following set: $\bar{U}^2 \triangleq \{(z_1,z_2) \in C \times C : |z_1| \le 1, |z_2| \le 1\}$, the 2-D system (1) is open-loop asymptotically stable or (A_1, A_2) is an asymptotically stable pair, if and only if

$$rank(I_n - z_1 A_1 - z_2 A_2) = n. \quad \forall (z_1,z_2) \in \bar{U}^2. \tag{6}$$

3 Main Results

3.1 Observer for Same Fault Sources

When the same fault appears in state equation and measurement equation, the system description becomes:

$$\begin{cases} x(i+1,j+1) = A_1 x(i,j+1) + A_2 x(i+1,j) \\ +B_1 u(i,j+1) + B_2 u(i+1,j) \\ +M_1 f(i,j+1) + M_2 f(i+1,j) \\ y(i,j) = Cx(i,j) + f(i,j), M_1, M_2 \in R^{n\times p} \end{cases} \tag{7}$$

Rewrite (7) into a descriptor system:

$$\begin{cases} E\bar{x}(i+1,j+1) = \bar{A}_{1m}\bar{x}(i,j+1) + \bar{A}_{2m}\bar{x}(i+1,j) \\ +\bar{B}_1 u(i,j+1) + \bar{B}_2 u(i+1,j) \\ y(i,j) = \bar{C}\bar{x}(i,j) \end{cases}$$

$$E = \begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \bar{A}_{1m} = \begin{bmatrix} A_1 & M_1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \bar{A}_{2m} = \begin{bmatrix} A_2 & M_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} C & I_p \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} B_1 \\ \mathbf{0} \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} B_2 \\ \mathbf{0} \end{bmatrix}, \tag{8}$$

$$\bar{x}(i+1,j+1) = \begin{bmatrix} x^T(i+1,j+1) & f^T(i+1,j+1) \end{bmatrix}^T$$

Theorem 1. There exist gain matrices L, K_1, $K_2 \in R^{(n+p)\times p}$ for the following observer:

$$\begin{cases} (E+L\bar{C})z(i+1,j+1) = F_{1m} z(i,j+1) \\ +F_{2m} z(i+1,j) + \bar{B}_1 u(i,j+1) + \bar{B}_2 u(i+1,j) \\ +G_{1m} y(i,j+1) + G_{2m} y(i+1,j), \\ \hat{\bar{x}}(i,j) = z(i,j) + (E+L\bar{C})^{-1} L y(i,j), \end{cases}$$

$$F_{1m} = \bar{A}_{1m} - K_1\bar{C}, F_{2m} = \bar{A}_{2m} - K_2\bar{C}, \tag{9}$$

$$G_{1m} = \bar{A}_{1m}(E+L\bar{C})^{-1}L, G_{2m} = \bar{A}_{2m}(E+L\bar{C})^{-1}L$$

with unknown boundary conditions

$$\begin{aligned} &\sup\|\hat{\bar{x}}(i,0)\| < \infty, i = 0,1,... \\ &\sup\|\hat{\bar{x}}(0,j)\| < \infty, j = 1,2,... \end{aligned} \tag{10}$$

such that $\lim_{i,j\to\infty}(\bar{x}(i,j) - \hat{\bar{x}}(i,j)) = \mathbf{0}$ or $\hat{\bar{x}}(i,j)$ is an asymptotic estimation of $\bar{x}(i,j)$, if and only if

$$rank\left\{ \begin{matrix} \begin{bmatrix} I_n - A_1 z_1 - A_2 z_2 & L_1 + K_{11} z_1 + K_{21} z_2 \\ \mathbf{0} & L_2 + K_{12} z_1 + K_{22} z_2 \end{bmatrix} \\ * \begin{bmatrix} I & \mathbf{0} \\ C & I \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -M_1 z_1 - M_2 z_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \end{matrix} \right\} \tag{11}$$

$$= n + p$$

Proof. L, K_1, K_2 can be partitioned as follows:

$$L = \begin{bmatrix} L_1^T & L_2^T \end{bmatrix}^T, K_1 = \begin{bmatrix} K_{11}^T & K_{12}^T \end{bmatrix}^T, K_2 = \begin{bmatrix} K_{21}^T & K_{22}^T \end{bmatrix}^T$$

where $L_1 \in R^{n\times p}$, $L_2 \in R^{p\times p}$, $K_{12} \in R^{p\times p}$, $K_{22} \in R^{p\times p}$, owing to

$$\begin{aligned} rank(E+L\bar{C}) &= rank(\begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}\begin{bmatrix} C & I_p \end{bmatrix}) \\ &= rank\begin{bmatrix} I_n + L_1 C & L_1 \\ L_2 C & L_2 \end{bmatrix} \\ &= rank\begin{bmatrix} I_n & L_1 \\ \mathbf{0} & L_2 \end{bmatrix} = rank(L_2) + n \end{aligned} \tag{12}$$

It is obvious that if and only if the square matrix L_2 is of full rank, $(E+L\bar{C})$ is nonsingular. Hence, letting L_2 be nonsingular, the following relationship can be obtained:

$$\begin{aligned} &\bar{C}(E+L\bar{C})^{-1}L \\ &= \begin{bmatrix} C & I_p \end{bmatrix} \begin{bmatrix} I_n & -L_1(L_2)^{-1} \\ -C & (I_p + CL_1)(L_2)^{-1} \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \\ &= \begin{bmatrix} C & I_p \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ I_p \end{bmatrix} = I_p. \end{aligned} \tag{13}$$

Adding $Ly(i+1, j+1)$ to both side of the first equation in (8) yields

$$
\begin{aligned}
&(E+L\bar{C})\bar{x}(i+1,j+1)=\bar{A}_{1m}\bar{x}(i,j+1)+\bar{A}_{2m}\bar{x}(i+1,j)\\
&+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)+Ly(i+1,j+1)\\
&=(\bar{A}_{1m}-K_1\bar{C})\bar{x}(i,j+1)+(\bar{A}_{2m}-K_2\bar{C})\bar{x}(i+1,j)\\
&+K_1y(i,j+1)+K_2y(i+1,j)\\
&+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)+Ly(i+1,j+1).
\end{aligned} \tag{14}
$$

Substituting $z(i,j)=\hat{\bar{x}}(i,j)-(E+L\bar{C})^{-1}Ly(i,j)$ into observer shown in (9) and according to the definition of $e(i,j)=\bar{x}(i,j)-\hat{\bar{x}}(i,j)$, the error dynamic of observer can be obtained as follows:

$$
\begin{aligned}
&e(i+1,j+1)=(E+L\bar{C})^{-1}(\bar{A}_{1m}-K_1\bar{C})e(i,j+1)\\
&+(E+L\bar{C})^{-1}(\bar{A}_{2m}-K_2\bar{C})e(i+1,j).
\end{aligned} \tag{15}
$$

Referring to Lemma 1, the error dynamic system (15) is asymptotically stable or $\lim_{i,j\to\infty} e(i,j)=\mathbf{0}$, if and only if

$$
rank\begin{bmatrix} I_{n+p}-z_1(E+L\bar{C})^{-1}(\bar{A}_{1m}-K_1\bar{C}) \\ -z_2(E+L\bar{C})^{-1}(\bar{A}_{2m}-K_2\bar{C}) \end{bmatrix}=n+p \tag{16}
$$

Notice that $\forall z_1,z_2\in\bar{U}^2$

$$
\begin{aligned}
&rank\begin{bmatrix} I_{n+p}-(E+L\bar{C})^{-1}(\bar{A}_{1m}-K_1\bar{C})z_1 \\ -(E+L\bar{C})^{-1}(\bar{A}_{2m}-K_2\bar{C})z_2 \end{bmatrix}\\
&=rank\left[(E+L\bar{C})-(\bar{A}_{1m}-K_1\bar{C})z_1-(\bar{A}_{2m}-K_2\bar{C})z_2\right]\\
&=rank\begin{bmatrix} I_n-A_1z_1-A_2z_2+L_1C+K_{11}Cz_1+K_{21}Cz_2 & L_1-M_1z_1-M_2z_2+K_{11}z_1+K_{21}z_2 \\ L_2C+K_{12}Cz_1+K_{22}Cz_2 & L_2+K_{12}z_1+K_{22}z_2 \end{bmatrix}\\
&=rank\left\{\begin{bmatrix} I_n-A_1z_1-A_2z_2 & L_1+K_{11}z_1+K_{21}z_2 \\ \mathbf{0} & L_2+K_{12}z_1+K_{22}z_2 \end{bmatrix} * \begin{bmatrix} I & \mathbf{0} \\ C & I \end{bmatrix}+\begin{bmatrix} \mathbf{0} & -M_1z_1-M_2z_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\right\}
\end{aligned} \tag{17}
$$

Thus, if and only if (11) holds, observer (9) is an asymptotically stable observer for 2-D system presented in (7). Referring to Definition 1, this completes the proof.

Remark 1. It can obtain the asymptotic estimation of both system states and fault vector at the same time without any restriction of unknown fault vector in theorem 1. The system description in (7) is similar to the description of a class of unknown input systems in [18] and [19], but an additional asymptotic estimation of fault vector can be obtained from observer (9) comparing with the observers presented in [18] and [19].

Remark 2. If the gain matrix of f is not unit matrix but $M_3\in R^{p\times q}$ with a rank of m, the scenario $p\ge q=m$ is contained in this section, and the scenario $p\ge q>m$ or $m\le p<q$ can be considered to be contained in the next section. Because if $p\ge q>m$ or $m\le p<q$, the $q-m$ elements of fault vector f cannot be used by observer as the feedback information. Generally speaking, the $q-m$ elements of f can be considered as another fault vector. In next section, two fault vectors existing in system will be discussed.

3.2 Observer for Different Fault Sources

When two individual fault vectors $f_s(i,j)$ and $f(i,j)$ appear in measurement equation and state equation, respectively, the system description becomes:

$$
\begin{cases}
x(i+1,j+1)=A_1x(i,j+1)+A_2x(i+1,j)\\
+B_1u(i,j+1)+B_2u(i+1,j)\\
+M_1f(i,j+1)+M_2f(i+1,j),\\
y(i,j)=Cx(i,j)+f_s(i,j),
\end{cases} \tag{18}
$$

$M_1,M_2\in R^{n\times q}$.

Rewriting (18) into the descriptor system form, it becomes

$$
\begin{cases}
E\bar{x}(i+1,j+1)=\bar{A}_{10}\bar{x}(i,j+1)+\bar{A}_{20}\bar{x}(i+1,j)\\
+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)\\
+\bar{M}_1f(i,j+1)+\bar{M}_2f(i+1,j)\\
y(i,j)=\bar{C}\bar{x}(i,j),
\end{cases}
$$

$$
E=\begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},\bar{A}_{10}=\begin{bmatrix} A_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},\bar{A}_{20}=\begin{bmatrix} A_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{19}
$$

$$
\bar{C}=\begin{bmatrix} C & I_p \end{bmatrix},\bar{B}_1=\begin{bmatrix} B_1 \\ \mathbf{0} \end{bmatrix},\bar{B}_2=\begin{bmatrix} B_2 \\ \mathbf{0} \end{bmatrix},
$$

$$
\bar{M}_1=\begin{bmatrix} M_1 \\ \mathbf{0} \end{bmatrix},\bar{M}_2=\begin{bmatrix} M_2 \\ \mathbf{0} \end{bmatrix}\in R^{(n+p)\times q}.
$$

where $\bar{x}(i+1,j+1)=\left[x(i+1,j+1) \quad f_s(i+1,j+1)\right]^T$.

Theorem 2. For 2-D system (19), there exists the gain matrices $L,K_1,K_2\in R^{(n+p)\times p}$ and $K_{d1},K_{d2}\in R^{q\times p}$ for the following observer:

$$
\begin{aligned}
&(E+L\bar{C})z(i+1,j+1)=F_{10}z(i,j+1)+F_{20}z(i+1,j)\\
&+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)\\
&+G_{10}y(i,j+1)+G_{20}y(i+1,j)\\
&+\bar{M}_1\hat{f}(i,j+1)+\bar{M}_2\hat{f}(i+1,j)\\
&\hat{f}(i+1,j+1)=A_{d1}\hat{f}(i,j+1)+A_{d2}\hat{f}(i+1,j)\\
&-K_{d1}\bar{C}z(i,j+1)-K_{d2}\bar{C}z(i+1,j)\\
&\hat{\bar{x}}(i,j)=z(i,j)+(E+L\bar{C})^{-1}Ly(i,j)\\
&F_{10}=\bar{A}_{10}-K_1\bar{C},F_{20}=\bar{A}_{20}-K_2\bar{C},\\
&G_{10}=\bar{A}_{10}(E+L\bar{C})^{-1}L,G_{20}=\bar{A}_{20}(E+L\bar{C})^{-1}L
\end{aligned} \tag{20}
$$

with the unknown boundary conditions

$$
\begin{aligned}
&\sup\left\|\hat{\bar{x}}(i,0)\right\|<\infty, i=0,1,...;\\
&\sup\left\|\hat{\bar{x}}(0,j)\right\|<\infty, j=1,2,...;\\
&\sup\left\|\hat{f}_a(i,0)\right\|<\infty, i=0,1,...;\\
&\sup\left\|\hat{f}_a(0,j)\right\|<\infty, j=1,2,...
\end{aligned} \tag{21}
$$

$\lim_{i,j\to\infty}(\bar{x}(i,j)-\hat{\bar{x}}(i,j))=\mathbf{0}$, $\lim_{i,j\to\infty}(f(i,j)-\hat{f}(i,j))=\mathbf{0}$ holds, if and only if

$$rank\begin{bmatrix} E+L\bar{C}-\bar{A}_{10}z_1-\bar{A}_{20}z_2+K_1\bar{C}z_1+K_2\bar{C}z_2 & -\bar{M}_1z_1-\bar{M}_2z_2 \\ K_{d1}\bar{C}z_1+K_{d2}\bar{C}z_2 & I_q-A_{d1}z_1-A_{d2}z_2 \end{bmatrix}=n+p+q \quad (22)$$

Proof. Adding $Ly(i+1,j+1)$ to both sides of the first equation in (19), it yields

$$\begin{aligned}&(E+L\bar{C})\bar{x}(i+1,j+1)=\bar{A}_{10}\bar{x}(i,j+1)\\&+\bar{A}_{20}\bar{x}(i+1,j)+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)\\&+\bar{M}_1f(i,j+1)+\bar{M}_2f(i+1,j)+Ly(i+1,j+1)\\&=(\bar{A}_{10}-K_1\bar{C})\bar{x}(i,j+1)+(\bar{A}_{20}-K_2\bar{C})\bar{x}(i+1,j)\\&+K_1y(i,j+1)+K_2y(i+1,j)\\&+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)+Ly(i+1,j+1)\\&+\bar{M}_1f(i,j+1)+\bar{M}_2f(i+1,j).\end{aligned} \quad (23)$$

Substituting $z(i,j)=\hat{\bar{x}}(i,j)-(E+L\bar{C})^{-1}Ly(i,j)$ into observer shown in (20), the observer in (20) comes to that

$$\begin{cases}(E+L\bar{C})\hat{\bar{x}}(i+1,j+1)=(\bar{A}_{10}-K_1\bar{C})\hat{\bar{x}}(i,j+1)\\+(\bar{A}_{20}-K_2\bar{C})\hat{\bar{x}}(i+1,j)+\bar{B}_1u(i,j+1)\\+\bar{B}_2u(i+1,j)+K_1y(i,j+1)+K_2y(i+1,j)\\+Ly(i+1,j+1)+\bar{M}_1\hat{f}(i,j+1)+\bar{M}_2\hat{f}(i+1,j).\\\hat{f}(i+1,j+1)=A_{d1}\hat{f}(i,j+1)+A_{d2}\hat{f}(i+1,j)\\+K_{d1}\bar{C}[\bar{x}(i,j+1)-\hat{\bar{x}}(i,j+1)]\\+K_{d2}\bar{C}[\bar{x}(i+1,j)-\hat{\bar{x}}(i+1,j)].\end{cases} \quad (24)$$

It comes the error dynamic system:

$$\begin{aligned}&(E+L\bar{C})e(i+1,j+1)=(\bar{A}_{10}-K_1\bar{C})e(i,j+1)\\&+(\bar{A}_{20}-K_2\bar{C})e(i+1,j)+\bar{M}_1e_d(i,j+1)\\&+\bar{M}_2e_d(i+1,j)\\&e_d(i+1,j+1)=A_{d1}e_d(i,j+1)+A_{d2}e_d(i+1,j)\\&-K_{d1}\bar{C}e(i,j+1)-K_{d2}\bar{C}e(i+1,j).\end{aligned} \quad (25)$$

Rewrite (25) into a compact form:

$$\underbrace{\begin{bmatrix}E+L\bar{C} & \mathbf{0}\\ \mathbf{0} & I_q\end{bmatrix}}_{\tilde{E}_L}\underbrace{\begin{bmatrix}e(i+1,j+1)\\ e_d(i+1,j+1)\end{bmatrix}}_{\tilde{e}(i+1,j+1)}=$$

$$\left(\underbrace{\begin{bmatrix}\bar{A}_{10} & \bar{M}_1\\ \mathbf{0} & A_{d1}\end{bmatrix}}_{\tilde{A}_{10}}-\underbrace{\begin{bmatrix}K_1\\ K_{d1}\end{bmatrix}}_{\tilde{K}_1}\underbrace{\begin{bmatrix}\bar{C} & \mathbf{0}\end{bmatrix}}_{\tilde{C}}\right)\underbrace{\begin{bmatrix}e(i,j+1)\\ e_d(i,j+1)\end{bmatrix}}_{\tilde{e}(i,j+1)}$$

$$+\left(\underbrace{\begin{bmatrix}\bar{A}_{20} & \bar{M}_2\\ \mathbf{0} & A_{d2}\end{bmatrix}}_{\tilde{A}_{20}}-\underbrace{\begin{bmatrix}K_2\\ K_{d2}\end{bmatrix}}_{\tilde{K}_2}\underbrace{\begin{bmatrix}\bar{C} & \mathbf{0}\end{bmatrix}}_{\tilde{C}}\right)\underbrace{\begin{bmatrix}e(i+1,j)\\ e_d(i+1,j)\end{bmatrix}}_{\tilde{e}(i+1,j)} \quad (26)$$

or

$$\begin{aligned}&\tilde{e}(i+1,j+1)=(\tilde{E}_L)^{-1}(\tilde{A}_{10}-\tilde{K}_1\tilde{C})\tilde{e}(i,j+1)\\&+(\tilde{E}_L)^{-1}(\tilde{A}_{20}-\tilde{K}_2\tilde{C})\tilde{e}(i+1,j).\end{aligned} \quad (27)$$

Similarly to the previous conclusion in Theorem 1, the error dynamic system (27) is asymptotically stable, if and only if

$$rank\begin{bmatrix}I_{n+p+q}-(\tilde{E}_L)^{-1}(\tilde{A}_{10}-\tilde{K}_1\tilde{C})z_1\\ -(\tilde{E}_L)^{-1}(\tilde{A}_{20}-\tilde{K}_2\tilde{C})z_2\end{bmatrix}=n+p+q \quad (28)$$

So it comes that

$$\begin{aligned}&rank\begin{bmatrix}I_{n+p+q}-(\tilde{E}_L)^{-1}(\tilde{A}_{10}-\tilde{K}_1\tilde{C})z_1\\ -(\tilde{E}_L)^{-1}(\tilde{A}_{20}-\tilde{K}_2\tilde{C})z_2\end{bmatrix}\\&=rank\begin{bmatrix}\tilde{E}_L-(\tilde{A}_{10}-\tilde{K}_1\tilde{C})z_1-(\tilde{A}_{20}-\tilde{K}_2\tilde{C})z_2\end{bmatrix}\\&=rank\begin{bmatrix}E+L\bar{C}-\bar{A}_{10}z_1-\bar{A}_{20}z_2+K_1\bar{C}z_1+K_2\bar{C}z_2 & -\bar{M}_1z_1-\bar{M}_2z_2\\ K_{d1}\bar{C}z_1+K_{d2}\bar{C}z_2 & I_q-A_{d1}z_1-A_{d2}z_2\end{bmatrix}\end{aligned} \quad (29)$$

if and only if (22) holds, observer (20) is an asymptotically stable observer for 2-D system presented in (18). it can be known that $\lim_{i,j\to\infty}(\bar{x}(i,j)-\hat{\bar{x}}(i,j))=\mathbf{0}$ and $\lim_{i,j\to\infty}(f(i,j)-\hat{f}(i,j))=\mathbf{0}$. Referring to Definition 1, it completes the proof.

Remark 3 According to the given 2-D system, choose appropriate K_1 and K_2 to meet (11) and (22). In fact, according to [2] (Theorem 1) or [20] (Theorem 4), one can find that the design of K_1 and K_2 can be formulated into a linear matrix inequality (LMI) problem after choosing L and checking the conditions in Theorem 1 and 2. Compute all the observer matrices with the chosen L, K_1, K_2, then 2-D asymptotically stable observers can be established.

4 Numerical Example

Example 1. Consider the 2-D FM-II system, where the system coefficient matrices and fault are as follows.

$$\begin{aligned}&A_1=\begin{bmatrix}0.4 & 2\\ 0 & 0\end{bmatrix},A_2=\begin{bmatrix}0 & 1\\ 0 & -2.4\end{bmatrix},B_1=\begin{bmatrix}1\\ 0\end{bmatrix},\\&B_2=\begin{bmatrix}0\\ 1\end{bmatrix},M_1=\begin{bmatrix}1\\ 0\end{bmatrix},M_2=\begin{bmatrix}0\\ -3\end{bmatrix},\\&C=\begin{bmatrix}0 & 1\end{bmatrix},f(i,j)=\sin(10(i-j)).\end{aligned} \quad (30)$$

The boundary conditions of the observer in this example are assumed to be zero and the boundary conditions of error dynamic system are equals to the boundary conditions of the original 2-D system. In the following examples, the boundary conditions of error is:

$$\begin{aligned}&e_1(0,j)=\sin(2\pi j/l),e_1(i,0)=\sin(\pi j/l),\\&e_2(0,j)=\sin(2\pi j/l),e_2(i,0)=\sin(2\pi j/l),\\&e_3(0,j)=2*\sin(2\pi j/l),e_3(i,0)=\sin(2\pi j/l),\\&0\le i,j\le l=40.\end{aligned} \quad (31)$$

It can be obtained that the condition in Theorem 1 is satisfied. Then the observer proposed in Theorem 1 can be designed for the 2-D system (30) with the coefficient matrices

$$L=\begin{bmatrix}0\\0\\1\end{bmatrix},K_1=\begin{bmatrix}10\\0\\0\end{bmatrix},K_2=\begin{bmatrix}0\\-3\\-0.9\end{bmatrix} \quad (32)$$

The obtained estimation error $e(i,j)\,(0\le i,j\le 40)$ is shown in Fig. 1. $e_1(i,j)$ and $e_2(i.j)$ are about the system states, and $e_3(i,j)$ is about the fault f. From Fig. 1 we can see that all the estimation errors go to zero asymptotically.

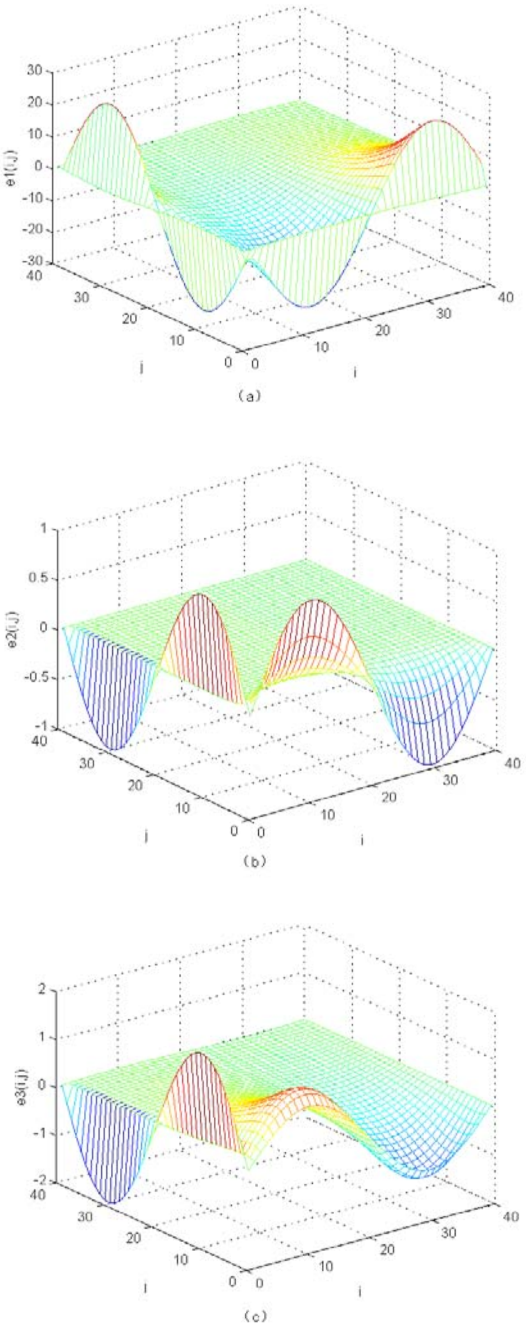


Fig.1. Estimation error $e(i,j)$ of the observer in Theorem 1 for 2-D system presented in (30). $e_1(i,j)$ and $e_2(i.j)$ are about the system states, and $e_3(i,j)$ is about the fault f.

Example 2. Consider the 2-D FM-II system, where the system coefficient matrices and faults are presented as follows:

$$A_1=\begin{bmatrix}0.5 & 0\\ 2 & 0\end{bmatrix}, A_2=\begin{bmatrix}0 & 0\\ 1.5 & 0.7\end{bmatrix}, B_1=\begin{bmatrix}1\\ 0\end{bmatrix}, B_2=\begin{bmatrix}0\\ 1\end{bmatrix}$$

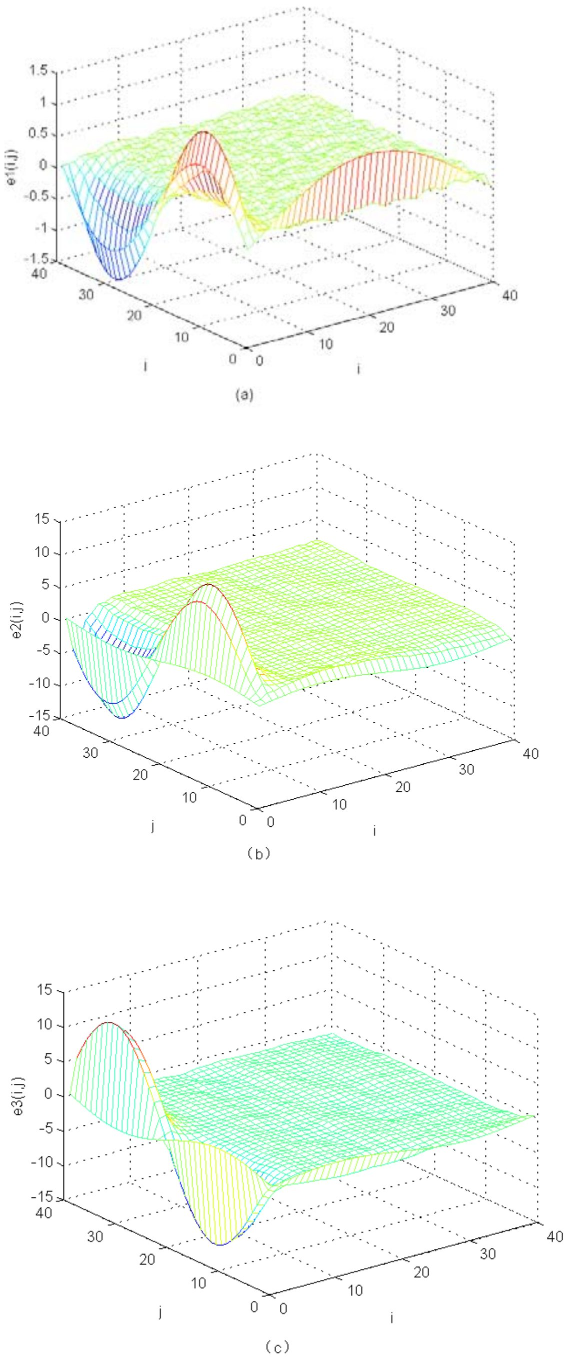
$$C=\begin{bmatrix}0 & 1\end{bmatrix}, f_s(i,j)=\sin(10(i-j)), M_1=M_2=\begin{bmatrix}0\\ 1\end{bmatrix} \tag{33}$$

$$f(i+1,j+1)=0.3f(i,j+1)-0.4f(i+1,j)$$

Then the observer proposed in Theorem 2 can be designed for the 2-D system presented in (33) with the observer coefficient matrices

$$L=\begin{bmatrix}0\\ 0\\ 100\end{bmatrix}, K_1=\begin{bmatrix}0\\ 0\\ 10\\ 0\end{bmatrix} K_2=\begin{bmatrix}0\\ 0\\ 10\\ 0\end{bmatrix}, K_{d1}=1, K_{d2}=1 \tag{34}$$

The obtained estimation error $e(i,j)\,(0\le i,j\le 40)$ is shown in Fig. 2. $e_1(i,j)$ and $e_2(i.j)$ are about the system states, $e_3(i,j)$ is about the fault f_s, and $e_4(i,j)$ is about the fault f. From Fig. 2 we can see that all the estimation errors do go to zero asymptotically.



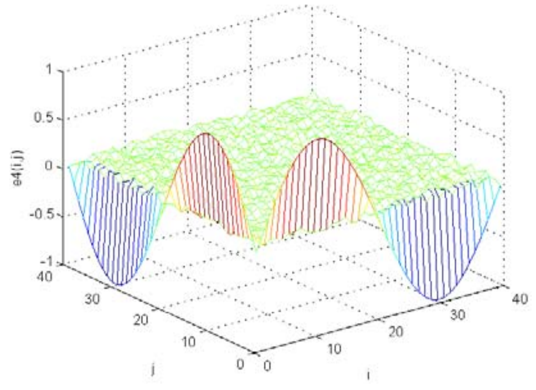


Fig.2. Estimation error $e(i, j)$ of the observer in Theorem 2 for 2-D system presented in (33). $e_1(i, j)$ and $e_2(i.j)$ are about the system states, $e_3(i, j)$ is about the fault f_s, and $e_4(i, j)$ is about the fault f.

5 Conclusions

In this paper, we study the simultaneous estimation of faults in 2-D systems. For the same fault occurs in in measurement equation and state equation, and two different types of faults respectively occur in measurement equation and state equation, the necessary and sufficient conditions for the existence of asymptotically stable observers are presented. Several numerical examples have been provided to demonstrate the feasibility and effectiveness of the proposed method. On the basis of the proposed observers, fault diagnosis and fault-tolerant control will be investigated.

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