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Fault-Tolerant Control of Stochastic Systems with Intermittent Faults and Time-Varying Delays

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Abstract: This paper investigates the fault-tolerant control of stochastic systems with intermittent faults and time-varying delays. The intermittent faults in multiplicative form occur in sensor and actuator simultaneously. Markov chain is used to depict the switching between the faulty and healthy situations. By designing a set of dynamic output feedback controllers, the resulted system is described by a Markovian jump system (MJS). The aim of this study is to stabilize the closed-loop system and make it satisfy the prescribed H_∞ performance index. Based on H_∞ control theory and linear matrix inequality (LMI) method, a sufficient condition for the existence of controller is given in this study.

Key Words: intermittent fault-tolerant control, Markov chain, H_∞ control, sensor and actuator faults

1 Introduction

Fault can be categorized as permanent fault (PF) and intermittent fault (IF). Permanent fault is commonly studied in the field of fault diagnosis (FD) and fault-tolerant control (FTC), permanent fault exists permanently if no corrective action is applied. Intermittent fault can disappear and occur without any intervention. Compared with the permanent fault, intermittent fault include special properties such as intermittence, repeatability and randomness[1-3]. Since the fault-tolerant control have been proposed in the 1970s, the majority of research is all about permanent fault in active fault-tolerant method[4-6] and passive fault-tolerant method[7-8]. However, in some practical systems, such as electronic circuit[9], communications equipment[10], mechanical device, aerospace aircraft[11] and high-speed rail, intermittent fault widely exists and seriously affects the performance of the system. To the authors' best knowledge, there are only few studies about intermittent fault and its fault tolerant control[12-13].

The most remarkable characteristic of intermittent fault is intermittence, it means that the system subject to faults will switch repeatedly between fault state and normal state. Traditional modeling method only considers the permanent fault and cannot be used to depict dynamic characteristics of intermittent fault. Because the occurrence and disappearance of intermittent fault are random, random variables are used to depict intermittent fault. For example, the random variable that satisfied Bernoulli distribution is used in [13] to model the multiplicative sensor intermittent faults. Based on the theory of dissipation, [11] studies the actuator intermittent fault in spacecraft attitude, and the Markov chain is employed to depict dynamic characteristics of the intermittent faults.

Time delays[14-17] are intrinsic properties of practical systems and probably lead to instability. Due to its importance and practicality, it has attracted persistent research attention in past years. Some initial results on Markovian jump system with time-varying delays have been derived. However, there are few studies that deal with intermittent fault and time delay simultaneously. Therefore, this study proposes a more realistic formulation for systems with multiple intermittent fault.

It is not hard to find that the present researches about intermittent fault-tolerant control mostly consider the single fault type (in sensor or actuator), or suppose that there is no time delay in systems. However, in the actual systems, sensor and actuator faults may appear at the same time, the time delays are existed widely. Based on the above two facts, this paper studies the FTC problem for time-varying delays stochastic systems with sensor and actuator intermittent faults. By augmenting states, the resulted system is described by a Markov jump system.

The rest of paper is organized as follows. Section 2 gives a new mathematical model of multiple IFs and some preliminary knowledge. Section 3 provides the main results of the proposed FTC scheme for time-varying delay systems with multiple sensor and actuator faults. Finally, conclusions are drawn in section 4.

Notations: Throughout the paper, I and $\mathbf{0}$ denote the identity matrix and zero matrix with appropriate dimension. $\mathbf{R}^n$ means n-dimensional space, $\mathbf{R}^{n\times m}$ represents all $n\times m$ real matrices. The superscript T denotes the matrix transpose, and the symbol $*$ indicates the corresponding transposed block in the symmetric block matrix. $diag\{\cdots\}$ and $\mathbb{E}[\bullet]$ represents a block diagonal matrix and mathematical expectation, respectively. $l_2[0, \infty)$ is the space of square-summable infinite sequence.

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2 Problem formulation and preliminaries

Consider a system subject to multiple sensor and actuator intermittent faults as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau(t)) \\ \qquad + B_1 M_A(t) u(t) + +D_1 w(t) \\ y(t) = M_S(t) Cx(t) \\ \qquad + C_d x(t-\tau(t)) + D_2 w(t) \\ z(t) = Ex(t) + E_d x(t-\tau(t)) \\ \qquad + B_3 u(t) + D_3 w(t) \\ x(t) = \ell(t) \forall t \in [-\bar{\tau}, 0] \end{cases} \tag{1}$$

$y(t) \in \mathbf{R}^p$, $z(t) \in \mathbf{R}^r$, $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$ are the measured outputs, desired controlled outputs, state and control input, respectively. $A, B_1, B_3, C, D_1, D_2, D_3, E,$ A_d, C_d, E_d are given matrices. $\tau(t)$ is the time-varying delay and satisfies:

$$0 < \tau(t) < \bar{\tau} < \infty, \dot{\tau}(t) < h \tag{2}$$

In the proposed description, the considered intermittent faults can be reflected in the terms:

$$\begin{cases} M_s(t) = diag\{m_{s1}, m_{s2}, \cdots m_{sp}\} \\ M_a(t) = diag\{m_{a1}, m_{a2}, \cdots m_{am}\} \end{cases}$$

m_i takes value within $[0,1]$. Different values means different fault size. For example, suppose that system involves four modes, mode 1 indicates that two sensors and two actuators are normal, and its matrix is $M_s(1) = diag\{1,1\}, M_a(1) = diag\{1,1\}$, mode 2 indicates that the first and second sensors/actuators are partially and absolutely faulty, and its matrix is $M_s(2) = diag\{0.5, 0\}, M_a(2) = diag\{0.5, 0\}$, mode 3 indicates that the first and second sensors are absolutely and partially faulty, each actuator is normal, and its matrix is $M_s(3) = diag\{0, 0.5\}, M_a(3) = diag\{1,1\}$, mode 4 indicates that each sensor is normal, the first and second actuators are absolutely and partially faulty, and its matrix is $M_s(4) = diag\{1,1\}, M_a(4) = diag\{0, 0.5\}$.

The study aims to design mode-dependent output feedback controllers with the following form:

$$\begin{cases} \dot{x}_c(t) = A_{c,i} x_c(t) + B_{c,i} y(t) \\ u(t) = C_{c,i} x_c(t) \end{cases} \tag{3}$$

$x_c(k) \in \mathbf{R}^n$ is the state of controller, $A_{c,i}, B_{c,i}, C_{c,i}$ are the controller matrices to be designed.

Augment states and define

$$\eta(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} \tag{4}$$

Applying controller (3) to system (1), the resulted MJS can be obtained as follows:

$$\begin{cases} \dot{\eta}(t) = \bar{A}_{ci}(t)\eta(t) + \bar{A}_{cdi}(t)\eta(t-\tau(t)) \\ \qquad + \bar{D}_{c1i} w(t) \\ z(t) = \bar{E}_{ci}\eta(t) + \bar{E}_{cdi}\eta(t-\tau(t)) \\ \qquad + \bar{D}_{c3i} w(t) \end{cases} \tag{5}$$

where

$$\bar{A}_{ci}(t) = \begin{bmatrix} A & B_1 M_A(t) C_{c,i} \\ B_{c,i} M_S(t) C & A_{c,i} \end{bmatrix}$$

$$\bar{A}_{cdi}(t) = \begin{bmatrix} A_d & 0 \\ B_{c,i} C_d & 0 \end{bmatrix}, \bar{E}_{cld,i} = \begin{bmatrix} E_d & 0 \end{bmatrix}$$

$$\bar{F}_{ci} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \bar{F}_{cdi} = \begin{bmatrix} F_d \\ 0 \end{bmatrix}, \bar{D}_{c3i} = D_3$$

$$\bar{D}_{c1i} = \begin{bmatrix} D_1 \\ B_{c,i} D_2 \end{bmatrix}, \bar{E}_{ci} = \begin{bmatrix} E & B_3 C_{c,i} \end{bmatrix}$$

The Markov chain takes values in set $\delta = \{1, \cdots, N\}$, an N mode Markov chain $\Lambda(t)$ containing N fault matrices is used to describe the N intermittent faults occurred in the sensors and actuators. Its transition probabilities matrix $P = (p_{ij})_{N \times N}$ has the following definitions:

$$p(\Lambda(t+\Delta) = j \mid \Lambda(t) = i) = \begin{cases} p_{ij}\Delta + o(\Delta), i \neq j \\ 1 + p_{ii}\Delta + o(\Delta), i = j \end{cases}$$

where $p_{ij} \geq 0 (i \neq j), p_{ii} = -\sum_{j=1, j\neq i}^{N} p_{ij}$.

Definition 1[15]: For the MJS:

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-\tau(t)) \\ x(t) = \ell(t) \qquad \forall t \in [-\bar{\tau}, \ 0] \end{cases} \tag{6}$$

The initial conditions are $\ell(t) \in \mathbf{R}^n$ defined on $[-\bar{\tau}, \ 0]$ and i_0 , if at time t , the following formula is satisfied

$$\lim_{t\to\infty} \mathbb{E}\left\{\int_0^t x^T(t) x(t) dt \middle| \ell(t), i_0\right\} < \infty \tag{7}$$

Then, (6) is said to be stochastically stable.

Definition 2[14]: Given scalar γ , if under zero initial condition, $\mathbb{E}\left\{\int_0^\infty z^T(t) z(t) dt\right\} \leq \gamma^2 \int_0^\infty w^T(t) w(t) dt$ holds for all nonzero $w(t) \in L_2[0, \infty)$. System (5) is stochastically stable with a H_∞ performance index γ .

Lemma 1[18]: (Schur complement) Given constant matrices $S_1 = S_1^T, S_2 = S_2^T > 0, S_3$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if :

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0$$

Lemma 2[17]: Given a scalar $\bar{\tau} > 0$. For time-varying delay $\tau(t)$ satisfying (2) and $P_i > 0, Q_i > 0, Y_i, W_i, Q > 0, Z > 0$, if the following LMIs hold:

$$\begin{bmatrix} \mathbf{\Omega}_i & \mathbf{\Psi}_i & \bar{\tau} Y_i & \bar{\tau} A_i^T Z \\ * & \Xi_i & \bar{\tau} W_i & \bar{\tau} A_{di}^T Z \\ * & * & -\bar{\tau} Z & 0 \\ * & * & * & -\bar{\tau} Z \end{bmatrix} < 0 \tag{8}$$

$$\sum_{j=1}^{N}\pi_{ij}Q_j \le Q \tag{9}$$

$$\mathbf{\Omega}_i=\sum_{j=1}^{N}\pi_{ij}P_j+P_iA_i+A_i^TP_i-Y_i-Y_i^T+Q_i+\bar{\tau}Q$$

$$\mathbf{\Psi}_i=P_iA_{di}+Y_i-W_i^T, \Xi_i=W_i+W_i^T-(1-h)\mathrm{Q}_i$$

Then, the MJS (6) is stochastically stable.

3 Main results

Theorem 1: Given scalar $\bar{\tau}>0$. For time-varying delay $\tau(t)$ satisfying (2) and matrices $Y_i, W_i, Q>0, \mathrm{Q}_i>0$, symmetric matrices $Z>0, P_i>0$, if the following LMIs hold:

$$\begin{bmatrix} \mathbf{\Omega}_i & \mathbf{\Psi}_i & \bar{\tau}Y_i & P_i\bar{D}_{c1i} & \bar{E}_{ci}^T & \bar{A}_{ci}^T \\ * & \Xi_i & \bar{\tau}W_i & 0 & \bar{E}_{cdi}^T & \bar{A}_{cdi}^T \\ * & * & -\bar{\tau}Z & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{D}_{c3i}^T & \bar{D}_{c1i}^T \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\bar{\tau}^{-1}Z^{-1} \end{bmatrix}<0 \tag{10}$$

$$\sum_{j=1}^{N}\pi_{ij}Q_j \le Q \tag{11}$$

Then, the system (5) is satisfied with definitions 1 and 2.

Proof: Comparing Theorem 1 with Lemma 2, one can obtain MJS (5) is stochastically stable. Choose the following stochastic Lyapunov function:

$$V(\eta_t,i,t)=V_1(\eta_t,i)+V_2(\eta_t,i,t)+V_3(\eta_t,i,t) \tag{12}$$

Where

$$V_1(\eta_t,i)=\eta(t)^T P_i\eta(t)$$
$$V_2(\eta_t,i,t)=\int_{t-\tau(t)}^{t}\eta(\alpha)^T Q_i\eta(\alpha)d\alpha$$
$$V_3(\eta_t,i,t)=\int_{-\bar{\tau}}^{0}\int_{t+\beta}^{t}\dot{\eta}(\alpha)^T Z\dot{\eta}(\alpha) +\eta(\alpha)^T Q\eta(\alpha)d\alpha d\beta$$

Let Δ be the weak generator of $\{\eta_t,i\}$ and define

$$\Delta V(\eta_t,i)=\lim_{\Delta\to 0}\frac{1}{\Delta}\left\{\mathbb{E}\left[V(\eta_{t+\Delta},i+\Delta)\middle|\eta_t,i\right]-V(\eta_t,i)\right\}$$

Then

$$\begin{aligned}\Delta V(\eta_t,i,t)&\le \eta(t)^T\sum_{j=1}^{N}\pi_{ij}P_j\eta(t)+2\eta(t)^T P_i\dot{\eta}(t)\\ &+\int_{t-\tau(t)}^{t}\eta(\alpha)^T\left(\sum_{j=1}^{N}\pi_{ij}Q_j\right)\eta(\alpha)d\alpha+\eta(t)^T Q_i\eta(t)\\ &-(1-h)\eta(t-\tau(t))^T Q_i\eta(t-\tau(t))+\bar{\tau}\dot{\eta}(t)^T Z\dot{\eta}(t)\\ &-\int_{t-\tau(t)}^{t}\dot{\eta}(\alpha)^T Z\dot{\eta}(\alpha)d\alpha+\bar{\tau}\eta(t)^T Q\eta(t)\\ &-\int_{t-\tau(t)}^{t}\eta(\alpha)^T Q\eta(\alpha)d\alpha\end{aligned} \tag{13}$$

In order to evaluate the H_∞ performance, the index is introduced as follows:

$$J=\mathbb{E}\int_0^\infty\left[z^T(t)z(t)-\gamma^2w^T(t)w(t)\right]dt \tag{14}$$

Under the zero initial condition, $V(0)=0, V(\infty)\ge 0$

$$\begin{aligned}J&=\mathbb{E}\int_0^\infty\left[z^T(t)z(t)-\gamma^2w^T(t)w(t)\right]dt\\ &=\mathbb{E}\int_0^\infty\left[z^T(t)z(t)-\gamma^2w^T(t)w(t)+\Delta V(t)\right]dt\\ &\quad+\mathbb{E}\{V(0)\}-\mathbb{E}\{V(\infty)\}\\ &\le\int_0^\infty\left[z^T(t)z(t)-\gamma^2w^T(t)w(t)+\Delta V(t)\right]dt\end{aligned}$$

Hence, from (5) and (13), one obtains

$$\begin{aligned}&z^T(t)z(t)-\gamma^2w^T(t)w(t)+\Delta V(t)\\ &=\left(\bar{E}_{cl,i}\eta(t)+\bar{E}_{cld,i}\eta(t-\tau(t))+\bar{D}_{cl3,i}w(t)\right)^T\\ &\left(\bar{E}_{cl,i}\eta(t)+\bar{E}_{cld,i}\eta(t-\tau(t))+\bar{D}_{cl3,i}w(t)\right)\\ &-\gamma^2w^T(t)w(t)+\Delta V(t)=\zeta(t,\alpha)^T\theta_i\zeta(t,\alpha)\end{aligned}$$

where

$$\zeta(t,\alpha)=\begin{bmatrix}\eta(t)^T & \eta(t-\tau(t))^T & \dot{\eta}(\alpha)^T & w(t)^T\end{bmatrix}^T$$

$$\theta_i=\begin{bmatrix}\Omega_i+\bar{\tau}\bar{A}_{ci}^TZ\bar{A}_{ci} & \Psi_i+\bar{\tau}\bar{A}_{ci}^TZ\bar{A}_{cdi} & \bar{\tau}Y_i & \bar{\tau}\bar{A}_{ci}^TZ\bar{D}_{c1i}\\ * & \Xi_i+\bar{\tau}\bar{A}_{cdi}^TZ\bar{A}_{cdi} & \bar{\tau}W_i & \bar{\tau}\bar{A}_{cdi}^TZ\bar{D}_{c1i}\\ * & * & -\bar{\tau}Z & 0\\ * & * & * & \bar{\tau}\bar{D}_{c1i}^TZ\bar{D}_{c1i}\end{bmatrix}$$
$$+\begin{bmatrix}\bar{E}_{ci}^T\bar{E}_{ci} & \bar{E}_{ci}^T\bar{E}_{cdi} & 0 & \bar{E}_{ci}^T\bar{D}_{c3i}+P_i\bar{D}_{c1i}\\ * & \bar{E}_{cdi}^T\bar{E}_{cdi} & 0 & \bar{E}_{cdi}^T\bar{D}_{c3i}\\ * & * & 0 & 0\\ * & * & * & \bar{D}_{c3i}^T\bar{D}_{c3i}-\gamma^2 I\end{bmatrix}$$

θ_i can be rewritten as:

$$\theta_i=\begin{bmatrix}\Omega_i & \Psi_i & \bar{\tau}Y_i & P_i\bar{D}_{c1i}\\ * & \Xi_i & \bar{\tau}W_i & 0\\ * & * & -\bar{\tau}Z & 0\\ * & * & * & -\gamma^2 I\end{bmatrix}+\phi_i$$

Where

$$\phi_i=\begin{bmatrix}\bar{E}_{ci}^T\bar{E}_{ci}+\bar{\tau}\bar{A}_{ci}^TZ\bar{A}_{ci} & \bar{E}_{ci}^T\bar{E}_{cdi}+\bar{\tau}\bar{A}_{ci}^TZ\bar{A}_{cdi} & 0 & \bar{E}_{ci}^T\bar{D}_{c3i}+\bar{\tau}\bar{A}_{ci}^TZ\bar{D}_{c1i}\\ * & \bar{E}_{cdi}^T\bar{E}_{cdi}+\bar{\tau}\bar{A}_{cdi}^TZ\bar{A}_{cdi} & 0 & \bar{E}_{cdi}^T\bar{D}_{c3i}+\bar{\tau}\bar{A}_{cdi}^TZ\bar{D}_{c1i}\\ * & * & 0 & 0\\ * & * & * & \bar{D}_{c3i}^T\bar{D}_{c3i}+\bar{\tau}\bar{D}_{c1i}^TZ\bar{D}_{c1i}\end{bmatrix}$$

ϕ_i can be rewritten as:

$$\phi_i=\begin{bmatrix}\bar{E}_{ci}^T & \bar{\tau}\bar{A}_{ci}^TZ\\ \bar{E}_{cdi}^T & \bar{\tau}\bar{A}_{cdi}^TZ\\ 0 & 0\\ \bar{D}_{c3i}^T & \bar{\tau}\bar{D}_{c1i}^TZ\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & \bar{\tau}^{-1}Z^{-1}\end{bmatrix}\begin{bmatrix}\bar{E}_{ci}^T & \bar{\tau}\bar{A}_{ci}^TZ\\ \bar{E}_{cdi}^T & \bar{\tau}\bar{A}_{cdi}^TZ\\ 0 & 0\\ \bar{D}_{c3i}^T & \bar{\tau}\bar{D}_{c1i}^TZ\end{bmatrix}^T$$
$$=\begin{bmatrix}\bar{E}_{ci}^T & \bar{A}_{ci}^T\\ \bar{E}_{cdi}^T & \bar{A}_{cdi}^T\\ 0 & 0\\ \bar{D}_{c3i}^T & \bar{D}_{c1i}^T\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & \bar{\tau}Z\end{bmatrix}\begin{bmatrix}\bar{E}_{ci}^T & \bar{A}_{ci}^T\\ \bar{E}_{cdi}^T & \bar{A}_{cdi}^T\\ 0 & 0\\ \bar{D}_{c3i}^T & \bar{D}_{c1i}^T\end{bmatrix}^T$$

Using Schur complement, one can directly get $\theta_i < 0$ and $J = \mathbb{E}\int_0^\infty \left[z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right]dt < 0$. Hence, the system (5) has a prescribed index γ. This completes the proof.

Theorem 2: Given scalar $\bar{\tau} > 0$. For time-varying delay $\tau(t)$ satisfying (2) and $\lambda_i, \kappa_i, \upsilon_i, \chi_i, \rho_i, \vartheta_i, \bar{\nu}_i, \varpi_i, \xi_i, \sigma_i, \Upsilon_i$, Φ_i, $Z > 0, Q > 0, Q_i > 0, Y_i, T_i, W_i$, if the following LMIs hold

$$\begin{bmatrix} H_{1i} & H_{2i} \\ H_{2i}^T & H_{3i} \end{bmatrix} < 0 \tag{15}$$

$$H_{1i} = \begin{bmatrix} \boldsymbol{\lambda}_i & \boldsymbol{\kappa}_i & \bar{\tau}T_i^T Y_i T_i & \boldsymbol{\chi}_i & \boldsymbol{\rho}_i & \bar{\boldsymbol{\nu}}_i \\ * & \boldsymbol{\upsilon}_i & \bar{\tau}T_i^T W_i T_i & 0 & \boldsymbol{\vartheta}_i & \boldsymbol{\varpi}_i \\ * & * & -\bar{\tau}T_i^T Z T_i & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D_{3i}^T & \boldsymbol{\sigma}_i \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\boldsymbol{\xi}_i \end{bmatrix}$$

$$H_{2i} = \begin{bmatrix} \bar{\tau}T_i^T & T_i^T & \Upsilon_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, H_{3i} = \begin{bmatrix} -\bar{\tau}Q^{-1} & 0 & 0 \\ * & -Q_i^{-1} & 0 \\ * & * & -\Phi_i^{-1} \end{bmatrix}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_j \le Q \tag{16}$$

Then, system (5) is satisfied with definitions 1 and 2. The controller parameters can be obtained as follows:

$$\begin{cases} A_{c,i} = \left(S_i^{-1} - X_i\right)^{-1}\left(M_i - X_i^T A S_i \right. \\ \qquad \left. - X_i^T B_1 M_a(t) L_i - F_i M_s(t) C S_i\right) S_i^{-1} \\ B_{c,i} = \left(S_i^{-1} - X_i\right)^{-1} F_i \\ C_{c,i} = L_i \left(S_i^{-1}\right)^T \end{cases} \tag{17}$$

Proof: In accordance with the approach of [16], P_i, T_i, Π_i can be partitioned as follows:

$$P_i = \begin{bmatrix} X_i & S_i^{-1} - X_i \\ \left(S_i^{-1} - X_i\right)^T & X_i - S_i^{-1} \end{bmatrix} > 0$$

$$T_i = \begin{bmatrix} S_i & I \\ S_i & \mathbf{0} \end{bmatrix}, \Pi_i = \begin{bmatrix} I & X_i \\ 0 & S_i^{-1} - X_i \end{bmatrix}$$

where all the blocks are $n \times n$ real symmetric matrices. One can verify that

$$\Pi_i = P_i T_i$$

$$T_i^T \sum\nolimits_{j=1}^{N} p_{ij} P_j T_i = \sum\nolimits_{j=1, j\ne i}^{N} p_{ij} \begin{bmatrix} S_i S_j^{-1} S_i & S_i S_j^{-1} \\ * & S_j^{-1} \end{bmatrix}$$

$$+ \begin{bmatrix} p_{ii} S_i & p_{ii} I \\ * & \sum\nolimits_{j=1}^{N} p_{ij} X_j - \sum\nolimits_{j=1, j\ne i}^{N} p_{ij} S_j^{-1} \end{bmatrix}$$

Besides, for any $Z \ge 0, \Pi_i^T Z^{-1} \Pi_i \ge \Pi_i^T + \Pi_i - Z$ is true. The relevant proof can be found in [16].

By applying the congruence transformation $diag\left\{T_i^T, T_i^T, T_i^T, I, I, \Pi_i^T\right\}$ to (10) and on the basis of Schur complement, one obtains the inequality (15). The block multiplications can be given as follows:

$$\boldsymbol{\lambda}_i = T_i^T \bar{\Omega}_i T_i = \bar{\boldsymbol{\nu}}_i + \bar{\boldsymbol{\nu}}_i^T + \Gamma_{2i} - T_i^T\left(Y_i + Y_i^T\right)T_i$$

$$\Gamma_{2i} = \begin{bmatrix} p_{ii} S_i & p_{ii} I \\ * & \sum\nolimits_{j=1}^{N} p_{ij} X_j - \sum\nolimits_{j=1, j\ne i}^{N} p_{ij} S_j^{-1} \end{bmatrix}$$

$$\boldsymbol{\kappa}_i = T_i^T \Psi_i T_i = \begin{bmatrix} A_d S_i & A_d \\ \mathbf{0} & X_i A_d + F_i C_d \end{bmatrix} + T_i^T\left(Y_i - W_i^T\right)T_i,$$

$$\boldsymbol{\upsilon}_i = T_i^T \Xi_i T_i = T_i^T\left(W_i + W_i^T - (1-h) Q_i\right)T_i$$

$$\boldsymbol{\chi}_i = T_i^T P_i \bar{D}_{c1i} = \begin{bmatrix} D_1 \\ X_i D_1 + F_i D_2 \end{bmatrix},$$

$$\bar{\boldsymbol{\nu}}_i = T_i^T \bar{A}_{ci}^T \Pi_i$$

$$= \begin{bmatrix} A S_i + B_1 M_a(t) L_i & A \\ M_i & X_i A + F_i M_s(t) C \end{bmatrix}$$

$$\boldsymbol{\varpi}_i = T_i^T \bar{A}_{cdi}^T \Pi_i = \begin{bmatrix} A_d S_i & A_d \\ \mathbf{0} & X_i A_d + F_i C_d \end{bmatrix}$$

$$\boldsymbol{\vartheta}_i = T_i^T \bar{E}_{cdi}^T = \begin{bmatrix} E_d S_i & E_d \end{bmatrix}$$

$$\boldsymbol{\rho}_i = T_i^T \bar{E}_{ci}^T = \begin{bmatrix} E S_i + B_3 L_i & E \end{bmatrix}$$

$$\boldsymbol{\sigma}_i = \bar{D}_{c1i}^T \Pi_i = \begin{bmatrix} D_1 \\ X_i D_1 + F_i D_2 \end{bmatrix}^T$$

$$\Phi_i = diag\left(S_1, \cdots, S_{i-1}, S_{i+1}, \cdots S_N\right)$$

$$\Upsilon_i = \begin{bmatrix} \sqrt{p_{i1}}\begin{bmatrix} S_i \\ I \end{bmatrix} & \cdots & \sqrt{p_{i(i-1)}}\begin{bmatrix} S_i \\ I \end{bmatrix} \cdots \\ \sqrt{p_{i(i+1)}}\begin{bmatrix} S_i \\ I \end{bmatrix} & \cdots & \sqrt{p_{iN}}\begin{bmatrix} S_i \\ I \end{bmatrix} \end{bmatrix}$$

$$\Pi_i^T\left(\bar{\tau}^{-1} Z^{-1}\right)\Pi_i \ge \bar{\tau}^{-1}\left(\Pi_i^T + \Pi_i - Z\right) = \boldsymbol{\xi}_i$$

where

$$L_i = C_{c,i} S_i^T, F_i = \left(S_i^{-1} - X_i\right) B_{c,i}$$

$$M_i = X_i^T A S_i + X_i^T B_1 M_a(t) L_i$$

$$+ F_i M_s(t) C S_i + \left(S_i^{-1} - X_i\right) A_{c,i} S_i$$

From the above formula, the controller parameters can then be derived as follows:

$$\begin{cases} A_{c,i} = \left(S_i^{-1} - X_i\right)^{-1}\left(M_i - X_i^T A S_i \right. \\ \qquad \left. - X_i^T B_1 M_a(t) L_i - F_i M_s(t) C S_i\right) S_i^{-1} \\ B_{c,i} = \left(S_i^{-1} - X_i\right)^{-1} F_i \\ C_{c,i} = L_i \left(S_i^{-1}\right)^T \end{cases}$$

This completes the proof.

Remark: In Theorem 2, the parameters of H_∞ dynamic output feedback controllers for time-delay system subject to multiple intermittent faults are given. It should be noted that (15) and (16) are not LMIs. Therefore, it is a nonconvex problem to solve (15) and (16). However, one can employ an iterative method used in [19] to solve this nonconvex problem and calculate the controller parameters.

4 Conclusions

In this paper, a fault-tolerant controller was designed for a class of stochastic systems with intermittent faults and time-varying delays. To approximate the actual situations, it is assumed that sensor and actuator faults may occur at the same time. The intermittent fault is described as Markov chain and then the closed-loop Markovian jump system model is obtained. Based on H_∞ control theory and linear matrix inequality (LMI) approach, a sufficient condition for the existence of fault-tolerant controller is derived.

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