

# Fault-tolerant Control for Nonlinear Systems with Multiple Intermittent Faults and Time-varying Delays

Liang Cao and Youqing Wang\*

**Abstract:** This study investigates a new fault-tolerant control method for uncertain nonlinear systems with multiple intermittent faults and time-varying delays. The considered intermittent faults appear in sensors and actuators simultaneously. A Markov chain is used to describe the random occurrence and disappearance of intermittent faults. The uncertain nonlinear system with intermittent faults is augmented as a Markovian jump system. By using H-infinity control theory and linear matrix inequality (LMI), we design fault tolerant controllers to make augmented Markovian jump system work steadily. Several sufficient conditions for stochastic stability with given H-infinity performance index and the existence of output-feedback controllers are derived. The effectiveness of the proposed fault-tolerant method is validated by a continuously stirred tank reactor (CSTR).

**Keywords:** Fault tolerant control,  $H_\infty$  control, intermittent faults, Markov model, time-varying delays.

## 1. INTRODUCTION

Most industrial processes contain intermittent faults (IFs) [1–9] and the faults usually cause great disruption in practical systems. As a special case of faults, IFs have been studied since the 1970's. They occur intermittently, remain in effect for a limited period of time, and can recover without any intervention [1]. Due to these properties, IFs are becoming challenging problems in electronics systems [2], network systems, aerospace aircraft [4], communication equipment [6], and high-speed rail systems. In past decades, some results addressing IF behavior have been studied in the fields of modeling [2], fault detection [7], and dynamics characterization [1]. According to [1,6], in practical devices and processes, IFs usually appear as weak noise and microbreaks in very early stages. As a function of time, Fig. 1 shows the amplitude of IFs increases and their effects become severe. Once IFs occur, their effects must be controlled as early as possible to prevent damage in the system safety and reliability.

Fault-tolerant control (FTC) [4, 5, 8–15] and fault detection [16–23] are proposed to guarantee a reliable and acceptable performance. A number of control theories, such as robust control [24, 25], predictive control [26], and fuzzy control [27, 28] are widely used. A traditional fault-tolerant control method usually deals with permanent faults. Compared with the permanent faults, the oc-

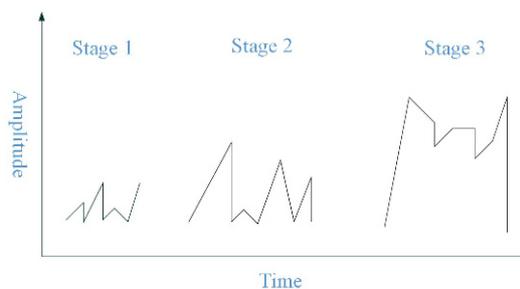


Fig. 1. The evolution of an intermittent fault.

currence of the IF exhibits special properties such as randomness, intermittence, and repeatability. Owing to the existence of these special properties, traditional fault diagnosis and fault tolerant control methods for addressing PFs cannot be applied to address IFs directly. From the viewpoint of FTC, the first hindrance is the lack of effective mathematical description. The existing intermittent fault model is mainly Bernoulli model. In [8], a Bernoulli distributed variable is used to model additive intermittent sensor and actuator faults. However, it is too simple and just can describe one fault mode and one normal mode. If a system subjects to many kinds of intermittent faults, the model would be no longer useful.

As typical stochastic systems, Markovian jump systems (MJSs) [29–32] have been extensively studied in modeling

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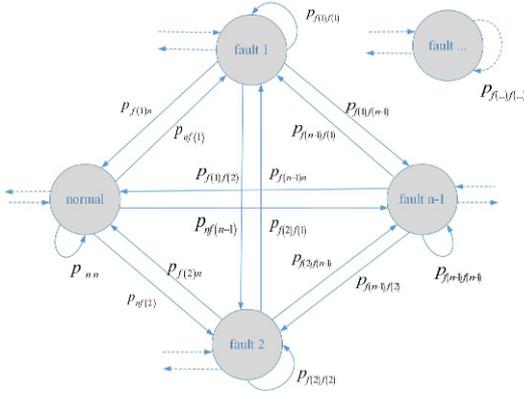


Fig. 2. An intermittent fault described by a  $N$ -mode Markovian jump system.

many processes where they may subject to abrupt changes in system parameters or structures. Considering intermittent nature of IFs, they can be seen as abrupt changes that frequently occur in different modes [33]. Hence, we propose a new IF Markov model that can combine IFs and MJSs together, and further discuss the intermittent feature of IFs by MJSs. In Fig. 2, when an IF occurs, the  $N$ -mode MJS abruptly changes from normal mode to faulty mode with a certain probability. If the IF disappears, the MJS returns to the normal mode with a certain probability. It can also switch among faulty modes and keep its mode with certain probability. Based on the new IF Markov model, one can transfer the fault -tolerant control of IFs into the stability analysis of MJS and some established results on MJSs can be applied to simplify the analysis.

Time delays [34–37] are intrinsic component of physical systems such as networked systems and chemical processes. Owing to its importance and practicality, it has attracted persistent research attention in previous studies. However, to the best knowledge of the authors, there is no reported study on a system with both IFs and time delays, which can affect each other and lead to poor performance and even instability. To describe more realistic processes, nonlinearity [38, 39] and parameter uncertainty [29] are considered in the paper. Because complete knowledge of transition probabilities in MJSs is difficult or costly to obtain in practice [40], it is assumed that transition probabilities are partly known. The main contributions of this study can be summarized as follows. According to the unique nature of intermittent faults, the authors first present a new and more reasonable model to describe the intermittent faults that occur in sensors and actuators simultaneously. Based on the new model, we use the reliable control method to achieve the fault tolerant control problem. Furthermore, to describe a more practical system, we first consider the time delays and parameter uncertainty in nonlinear systems that subject to intermittent sensor and actuator faults.

**Notations:**  $\mathbf{R}^n$  denotes the  $n$ -dimensional space, and  $\mathbf{R}^{n \times m}$  indicates the set of all  $n \times m$  real matrices.  $E[\bullet]$  is the mathematical expectation of variable  $\bullet$ .  $X > 0$  means that real symmetric matrix  $X$  is positive definite. Symbol  $*$  stands for the symmetric block matrices.  $\text{diag}\{\dots\}$  and  $l_2[0, \infty)$  represent a block diagonal matrix and the space of a square summable infinite sequence, respectively.  $I$  and  $\mathbf{0}$  are identity and zero matrices with compatible dimensions.

## 2. PROBLEM FORMULATION

### 2.1. System description

Consider the following uncertain nonlinear systems with time-varying delays and IFs:

$$\begin{cases} x(k+1) = (A + \Delta A(k))x(k) \\ \quad + (A_d + \Delta A_d(k))x(k - \tau(k)) \\ \quad + (F + \Delta F(k))f(x(k)) \\ \quad + B_1 M_A(k)u(k) \\ \quad + (F_d + \Delta F_d(k))f_d(x(k - \tau(k))) \\ \quad + D_1 w(k), \\ y(k) = M_S(k)[C x(k) + C_d x(k - \tau(k))] \\ \quad + D_2 w(k), \\ z(k) = E x(k), \\ x(k) = l(k) \quad \forall k \in [-\bar{\tau}, 0], \end{cases} \quad (1)$$

$x(k) \in \mathbf{R}^n$ ,  $y(k) \in \mathbf{R}^m$ ,  $u(k) \in \mathbf{R}^p$ ,  $w(k) \in \mathbf{R}^d$ , and  $z(k) \in \mathbf{R}^r$  are the system state, measured output, input, disturbance, and desired output, respectively.  $M_A(k)$ ,  $M_S(k)$  are intermittent fault matrices that are governed by the Markov model.  $A$ ,  $A_d$ ,  $F$ ,  $F_d$ ,  $B_1$ ,  $C$ ,  $C_d$ ,  $D_1$ ,  $D_2$  and  $E$  are known matrices with appropriate dimensions.  $\tau(k)$  denotes the delay satisfying  $\underline{\tau} \leq \tau(k) \leq \bar{\tau}$ , where  $l(-\bar{\tau}), \dots, l(0)$  are the initial conditions. The detailed block diagram of system (1) is given in Fig. 3.

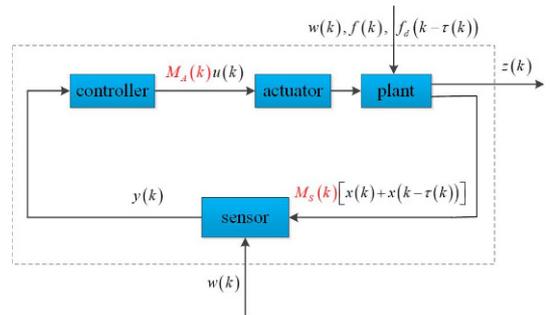


Fig. 3. System description with actuator and sensor intermittent faults.

## 2.2. Time-varying parameter uncertainties

Time-varying parameter uncertainties  $\Delta A(k)$ ,  $\Delta A_d(k)$ ,  $\Delta F(k)$ , and  $\Delta F_d(k)$  are matrices of the form:

$$\begin{aligned} & \begin{bmatrix} \Delta A(k) & \Delta A_d(k) & \Delta F(k) & \Delta F_d(k) \end{bmatrix} \\ & = M\Lambda(k) \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}, \end{aligned} \quad (2)$$

where  $M$ ,  $N_1$ ,  $N_2$ ,  $N_3$ ,  $N_4$  are given matrices.  $\Lambda(k)$  is the unknown time-varying matrix function satisfying  $\Lambda^T(k)\Lambda(k) \leq I$ .

## 2.3. Nonlinear functions

For vector-valued functions  $f(\cdot)$  and  $f_d(\cdot)$ , we assumed that

$$[f(x) - U_1(x)]^T [f(x) - U_2(x)] \leq 0, \forall x \in \mathbf{R}^n, \quad (3)$$

$$[f_d(x) - V_1(x)]^T [f_d(x) - V_2(x)] \leq 0, \forall x \in \mathbf{R}^n, \quad (4)$$

where  $U_1$ ,  $U_2$ ,  $V_1$ ,  $V_2$  are known matrices.  $U_1 - U_2$  and  $V_1 - V_2$  are positive definite matrices.

**Remark 1:** Note that the sector nonlinearities have been intensively studied, see e.g., [38, 41]. The nonlinear functions  $f$ ,  $f_d$  are said to belong to sectors. The descriptions in (3) and (4) are quite general, and the well-known Lipschitz conditions are the special case of this description. In what follows, for simplicity and without loss of generality, we always assume that  $f(0) = 0$ ,  $f_d(0) = 0$ .

**Remark 2:** It should be noticed that some general nonlinear systems that based on Takagi-Sugeno (T-S) fuzzy affine dynamic models [41] or piecewise affine dynamic models [42] can also be used in the future owing to their powerful identification and approximation ability to general smooth nonlinear systems.

## 2.4. Fault description

$M_A = \text{diag}\{m_{a1}, \dots, m_{ap}\}$  and  $M_S = \text{diag}\{m_{s1}, \dots, m_{sm}\}$  are used to describe actuator IFs and sensor IFs, respectively. The  $i$ -th element  $m_i$  on a diagonal matrix takes values within  $[0, 1]$  and different values mean different fault condition. For example, if  $M_A = \{0.5, 0\}$ , the first actuator is partially at fault and the second actuator is absolutely at fault. Likewise, if  $M_S = \{0, 1\}$ , the first sensor is absolutely at fault while the second sensor is fault free. We use a  $N$ -mode Markov chain  $\Xi(k)$  to describe  $N$  IFs. In these modes, only one stands for fault-free conditions in all sensors and actuators, the other modes indicate there are faults in sensors and/or actuators. The transition probability from mode  $i$  to mode  $j$  is presented as  $p_{ij} = p(\Xi(k+1) = j | \Xi(k) = i)$  and satisfies  $\sum_{j=1}^N p_{ij} = 1$ . Additionally, transition probabilities are assumed to be partly known in our description. For  $i \in \Psi = \{1, \dots, N\}$ ,  $\Psi_K^i := \{j : p_{ij} \text{ is known}\}$ ,  $\Psi_U^i := \{j : p_{ij} \text{ is unknown}\}$ ,  $\Psi = \Psi_K^i \cup \Psi_U^i$ .

**Remark 3:** The detailed description of our mathematical model can be found in Fig. 2 and Section 2.4.

Based on the new intermittent fault model, we transfer the fault-tolerant control of intermittent faults into the stability analysis of Markovian jump systems, and some established results on Markovian jump systems can be applied to enhance the study.

**Remark 4:** IFs exist widely in many situations such as net congestion and packet dropout in networked systems [43] and electromagnetic interference in electronic systems. The majority of IFs are activated and inactivated by themselves. Particularly, IFs are the major cause for circuit system failure [44]. We can apply the proposed FTC strategy to many practical systems such as aircraft [4], mechanical devices, distributed systems [45], and communication.

## 2.5. Feedback controller design

Considering the following mode-dependent output feedback controller:

$$\begin{aligned} x_c(k+1) &= A_{ci}x_c(k) + B_{ci}y(k), \\ u(k) &= C_{ci}x_c(k), \end{aligned} \quad (5)$$

where  $x_c(k) \in \mathbf{R}^c$  is the state of the controller and  $A_{ci}$ ,  $B_{ci}$ , and  $C_{ci}$  are the controller matrices to be designed. Define

$$\zeta(k) = \begin{bmatrix} x^T(k) & x_c^T(k) \end{bmatrix}^T, \quad (6)$$

and taking (1), (5), and (6) into consideration, the resulting MJS can be deduced as follows:

$$\begin{cases} \zeta(k+1) = \bar{A}_i\zeta(k) + \bar{A}_{di}T\zeta(k - \tau(k)) \\ \quad + \bar{F}_if(x(k)) + \bar{F}_{di}f_d(x(k - \tau(k))) \\ \quad + D_iw(k), \\ z(k) = \bar{E}_i\zeta(k), \end{cases} \quad (7)$$

where

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A & B_1M_A(k)C_{ci} \\ B_{ci}M_S(k)C & A_{ci} \end{bmatrix} + \bar{M}\Lambda(k)\bar{N}_1, \\ \bar{M} &= \begin{bmatrix} M \\ 0 \end{bmatrix}, \bar{N}_1 = \begin{bmatrix} N_1 & 0 \end{bmatrix}, \\ \bar{A}_{di} &= \begin{bmatrix} A_d \\ B_{ci}M_S(k)C_d \end{bmatrix} + \bar{M}\Lambda(k)\bar{N}_2, \\ T &= \begin{bmatrix} I & \mathbf{0} \end{bmatrix}, \bar{N}_2 = \begin{bmatrix} N_2 & 0 \end{bmatrix}, \\ \bar{F}_i &= F_i + \begin{bmatrix} \Delta F \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} F \\ \mathbf{0} \end{bmatrix} + \bar{M}\Lambda(k)\bar{N}_3, \\ D_i &= \begin{bmatrix} D_1 \\ B_{ci}D_2 \end{bmatrix}, \bar{N}_3 = \begin{bmatrix} N_3 & 0 \end{bmatrix}, \\ \bar{F}_{di} &= F_{di} + \begin{bmatrix} \Delta F_d \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} F_d \\ \mathbf{0} \end{bmatrix} + \bar{M}\Lambda(k)\bar{N}_4, \\ E_i &= \begin{bmatrix} E & \mathbf{0} \end{bmatrix}, \bar{N}_4 = \begin{bmatrix} N_4 & 0 \end{bmatrix}. \end{aligned} \quad (8)$$

Denote  $P$  as the transition probability matrix of resulted MJS (7). Some elements of the matrix  $P$  are assumed to be

unknown. It is worth pointing out that any of the elements can be unknown in each row as long as one known element exists. Then, for system (7) with  $N$  modes, one example of a transition probability matrix can be written as:

$$P = \begin{bmatrix} ? & p_{12} & \cdots & ? \\ ? & ? & \cdots & p_{2N} \\ \vdots & \vdots & ? & \vdots \\ p_{N1} & p_{N2} & \cdots & ? \end{bmatrix},$$

where "?" denotes unknown elements. It should be notice that any element could be unknown, but at least one known element exists in each row. That is to say, for a  $N \times N$  matrix, the number of unknown elements can be up to  $N^2 - N$ . The objective of this work is to design mode-dependent controller (5) for system (1), when the resulted closed-loop system (7) satisfies both of the following two requirements:

1) The resulted system (7) is stochastically stable in the case of  $w(k) = 0$ .

2) The resulted system (7) has a prescribed  $H_\infty$  performance index  $\gamma$ , i.e., under zero initial condition

$$\|z(k)\|_2^2 < \gamma^2 \|w(k)\|_2^2, \forall w(k) \in l_2(0, \infty], \quad (9)$$

where

$$\|z(k)\|_2^2 = \sum_{k=0}^{\infty} z^T(k)z(k),$$

$$\|w(k)\|_2^2 = \sum_{k=0}^{\infty} w^T(k)w(k).$$

The definition of stochastic stability is needed for further analysis.

**Definition 1 [35]:** For any Markov model initial condition  $r(0) \in \psi$  and system state initial condition  $\zeta(0) \in R^n$ , the resulted system (7) with  $w(k) = 0$  is stochastically stable if the following formula is satisfied:

$$\sum_{k=0}^{\infty} E \left( \|\zeta(k)\|^2 | \zeta(0), r(0) \right) < \infty. \quad (10)$$

To derive the main results, the following lemmas are used.

**Lemma 1 [39]:** Given constant matrices  $\Omega_1 = \Omega_1^T$ ,  $\Omega_2 = \Omega_2^T > 0$ ,  $\Omega_3$ ,  $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ , if and only if :

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

**Lemma 2 [29]:** Considering matrices  $\Omega$ ,  $M$ ,  $F$ ,  $N$  with appropriate dimensions and  $F^T F \leq I$ , for any scalar  $\varepsilon > 0$

$$\Omega + MFN + (MFN)^T \leq \Omega + \varepsilon^{-1}MM^T + \varepsilon N^T N.$$

**Lemma 3 [8]:** Let  $Z_0(x)$ ,  $Z_1(x)$ ,  $\dots$ ,  $Z_l(x)$  be quadratic functions and  $\Phi_i^T = \Phi_i$ , namely,  $Z_i(x) = x^T \Phi_i x$ . For  $\Phi_0 - \sum_{i=1}^l \tau_i \Phi_i < 0$ ,  $\tau_1 \geq 0$ ,  $\tau_2 \geq 0, \dots, \tau_l \geq 0$ , one can get  $Z_0(x) < 0$ , if and only if  $Z_1(x) \leq 0, \dots, Z_l(x) \leq 0$ .

### 3. MAIN RESULTS

**Theorem 1:** For the resulted system (7), given scalar  $\gamma > 0$ , if there exists symmetrical positive definite matrices  $P_i$ ,  $Q$ , matrices  $V_1, V_2, U_1, U_2$ , and constant scalars  $\mu, \lambda, \varepsilon, \tau, \bar{\tau}$ , such that the following matrices inequality hold:

$$\Upsilon_{jK} = \begin{bmatrix} \Upsilon_{1jK} & \Upsilon_{2jK} \\ * & \Upsilon_{3jK} \end{bmatrix} < 0, \quad \forall j \in \psi_K^i, \quad (11)$$

$$\Upsilon_{jU} = \begin{bmatrix} \Upsilon_{1jU} & \Upsilon_{2jU} \\ * & \Upsilon_{3jU} \end{bmatrix} < 0, \quad \forall j \in \psi_U^i, \quad (12)$$

then the resulted system (7) is stochastically stable and satisfies

$$\|z(k)\|_2^2 < \gamma^2 \|w(k)\|_2^2, \quad \forall w(k) \in l_2(0, \infty),$$

where

$$\sum_{j \in \psi_K^i} p_{ij} P_j \doteq P_K^j, \quad \sum_{j \in \psi_K^i} p_{ij} P_i \doteq p_K^i P_i,$$

$$\Upsilon_{3jK} = \begin{bmatrix} -P_K^j & \mathbf{0} \\ * & -p_K^i I \end{bmatrix}, \quad \Upsilon_{3jU} = \begin{bmatrix} -P_j & \mathbf{0} \\ * & -I \end{bmatrix},$$

$$\bar{\Pi}_2 = p_K^i (1 + \bar{\tau} - \tau) T^T Q T - p_K^i \mu T^T \bar{U}_1 T - p_K^i P_i,$$

$$\bar{\Pi}_{2U} = (1 + \bar{\tau} - \tau) T^T Q T - \mu T^T \bar{U}_1 T - P_i.$$

**Proof:** The proof can be divided into two parts. First, we need to prove that the output feedback controller (5) can stabilize the system (1) which subject to IFs. Because the disturbance  $w(k)$  does not affect the stability of system, we assume  $w(k) = 0$  in the first step. Using Lyapunov-Krasovskii function, we obtain that (10) holds, which means that the resulted system (7) is stochastically stable. Second, we analyze the robustness of system (7) in the existence of disturbance  $w(k)$ . In order to ensure its  $H_\infty$  performance, we substitute  $z(k) = \bar{E}_i \zeta(k)$  into  $\|z(k)\|_2^2$ . Because Theorem 1 is the sufficient condition, using (11) and (12), we get that  $\|z(k)\|_2^2 < \gamma^2 \|w(k)\|_2^2$ . That is to say, for uncertain nonlinear systems (7) with IFs, they can work steadily and have a satisfied performance index. In order to make the paper easy to read, we put the proof of the Theorem 1 in Appendix A.  $\square$

Up until now, we have obtained the sufficient condition for stochastic stability and the existence of output feedback controllers. However, because the existence of  $-\left( \sum_{j \in \psi_K^i} p_{ij} P_j + \sum_{j \in \psi_U^i} p_{ij} P_j \right)^{-1}$ , it shall be noticed that (11) is not a linear matrix inequality. So, Theorem 2 is obtained to get the LMI condition and concrete controller parameters.

**Theorem 2:** There exist fault-tolerant controllers of the form (5), positive symmetrical matrices  $P_i$ ,  $Q$ , and scalars  $\gamma > 0$ ,  $\mu > 0$ ,  $\lambda > 0$ ,  $\varepsilon > 0$  that satisfy (11), (12), if and

only if, for given symmetrical matrices  $Y_i$ , there exist symmetrical matrices  $X_i$ , matrices  $\Phi_{1i}$ ,  $\Phi_{2i}$ ,  $\Phi_{3i}$ , that satisfy the following LMIs:

$$\begin{bmatrix} \Psi_{1i} & \Psi_{2i} \\ * & \Psi_{3i} \end{bmatrix} < 0, \quad (13)$$

$$\begin{bmatrix} \tilde{Y}_{qi} & \hat{Y}_{qi}^T \\ * & Y_j \end{bmatrix} < 0, \quad (14)$$

where

$$\Psi_{1i} =$$

$$\begin{bmatrix} \vartheta_i & Y_i^T \vartheta - I & \mathbf{0} & -\mu Y_i^T \bar{U}_2 & \mathbf{0} & \mathbf{0} \\ * & \vartheta - X_i & \mathbf{0} & -\mu \bar{U}_2 & \mathbf{0} & \mathbf{0} \\ * & * & \bar{\omega} & \mathbf{0} & -\lambda \bar{V}_2 & \mathbf{0} \\ * & * & * & -\mu I & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\lambda I & \mathbf{0} \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\vartheta_i = Y_i^T \vartheta Y_i - Y_i, \bar{\omega} = -Q - \lambda \bar{V}_1,$$

$$\Psi_{2i} =$$

$$\begin{bmatrix} \tilde{\Phi}_{1i} & \Phi_{2i}^T & Y_i E^T & \mathbf{0} & Y_i N_1^T \\ A^T & A^T \hat{X}_i + C_d^T M_s(k)^T \Phi_{3i}^T & E^T & \mathbf{0} & N_1^T \\ A_d^T & A_d^T \hat{X}_i + C_d^T M_s(k)^T \Phi_{3i}^T & \mathbf{0} & \mathbf{0} & N_2^T \\ F^T & F^T \hat{X}_i & \mathbf{0} & \mathbf{0} & N_3^T \\ F_d^T & F_d^T \hat{X}_i & \mathbf{0} & \mathbf{0} & N_4^T \\ D_1^T & D_1^T \hat{X}_i + D_2^T \Phi_{3i}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\tilde{\Phi}_{1i} = Y_i A^T + \Phi_{1i}^T M_A(k)^T B_1^T,$$

$$\Psi_{3i} = \begin{bmatrix} \tilde{Y}_{qi} - \hat{Y}_{qi} - \hat{Y}_{qi}^T & -I & \mathbf{0} & M & \mathbf{0} \\ * & -\hat{X}_i & \mathbf{0} & \hat{X}_i^T M & \mathbf{0} \\ * & * & -I & \mathbf{0} & \mathbf{0} \\ * & * & * & -\varepsilon I & \mathbf{0} \\ * & * & * & * & -\varepsilon^{-1} I \end{bmatrix}.$$

Some definitions of relevant matrices in (13) and (14) can be found in Appendix B. Furthermore, the fault tolerant controller matrices  $A_{ci}$ ,  $B_{ci}$ ,  $C_{ci}$  are given as follows:

$$\begin{cases} A_{c,i} = (\hat{Y}_{qi}^{-1} - \hat{X}_i)^{-1} \\ \quad \times \left( \Phi_{2i} - \hat{X}_i^T A Y_i - \hat{X}_i^T B_1 M_A(k) \Phi_{1i} \right) Y_i^{-1}, \\ B_{c,i} = (\hat{Y}_{qi}^{-1} - \hat{X}_i)^{-1} \Phi_{3i}, \\ C_{c,i} = \Phi_{1i} Y_i^{-1}. \end{cases} \quad (15)$$

**Proof:** The proof can be divided into two parts. First, it contains uncertainties in (11) and (12) such as  $\Delta A$ ,  $\Delta F$ , we deal with the problem of time-varying parameter uncertainties by using Lemma 2. Second, we partitioned matrices and make some congruence transformations, then, we can calculate the controller parameters by some particular block multiplications. In order to make the paper easy to read, we put the proof of the Theorem 2 in Appendix B.  $\square$

**Remark 5:** Theorem 2 shows that the feasibility of the fault-tolerant control problem can be readily checked by the solvability of LMI (13) and (14), which can be determined by using the Matlab LMI toolbox in a straightforward way. Actually, once we get the given values of  $\bar{\tau}$ ,  $\underline{\tau}$ ,  $\gamma > 0$ ,  $\varepsilon > 0$ , and nonlinear function matrices, the solvability of LMI (13) and (14) have little computational burden, which means the computation complexity is very small and the proposed method can be utilized on-line. Summarizing Theorems 1 and 2, the output feedback controller design algorithm is proposed as follows:

**Algorithm 1:**

Step 1: Given time delay  $\bar{\tau}$ ,  $\underline{\tau}$ ,  $H_\infty$  performance index  $\gamma > 0$ , parameter uncertainty scalar  $\varepsilon > 0$ .

Step 2: Given nonlinear function matrices  $U_1$ ,  $U_2$ ,  $V_1$ ,  $V_2$ , choose fault matrices  $M_A(k)$ ,  $M_S(k)$  and partly known transition probability matrix  $P$ .

Step 3: Choose symmetrical matrices  $Y_i$ .

Step 4: Solve the LMIs (13) and (14).

Step 5: If the LMIs are feasible, calculate the controller parameters. If the LMI is infeasible, return to Step 3.

**Remark 6:** The proposed method can be extended to networked control systems (NCSs) [46]. The introduction of communication channels in the NCSs brings some network-induced critical issues or constraints such as variable transmission delays, data packet dropouts, packet disorder, and quantization errors, which would significantly degrade the system performance or even destabilize the system in certain conditions. Comparing with our method, the data packet dropouts can be seen as the occurrence of intermittent faults (except partly faulty), the reliable control and robust control methods in this paper can also be used in NCSs.

#### 4. ILLUSTRATIVE EXAMPLE

In this part, to show the effectiveness of the proposed method, it is tested on a typical nonlinear chemical process, a continuously stirred tank reactor (CSTR) [47, 48]. Fig. 4 gives a schematic description of a well-mixed CSTR with an isothermal, liquid phase, multi-component chemical reaction  $A \rightleftharpoons B \rightarrow C$ .

The dynamics of CSTR are modeled by the following equations:

$$\begin{cases} \dot{\bar{x}}_1 = 1 - \bar{x}_1 - Da_1 \bar{x}_1 - Da_2 \bar{x}_2^2, \\ \dot{\bar{x}}_2 = Da_1 \bar{x}_1 - \bar{x}_2 - Da_2 \bar{x}_2^2 - Da_3 \bar{x}_2^2 + \bar{u}, \\ \dot{\bar{x}}_3 = Da_3 \bar{x}_2^2 - \bar{x}_3, \\ \bar{z} = \bar{x}_3, \end{cases} \quad (16)$$

where

$$\bar{x}_1 = C_A/C_{AF}, \bar{x}_2 = C_B/C_{AF}, \bar{x}_3 = C_C/C_{AF}, \bar{u} = N_{BF}/F C_{AF}.$$

Detailed definitions of the parameters are given in Table 1.

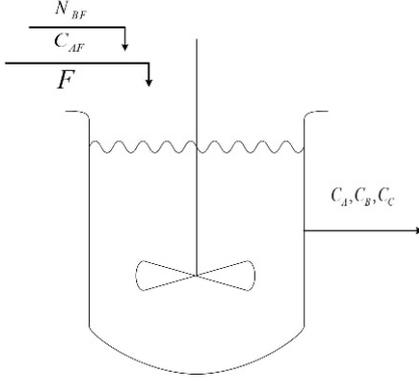


Fig. 4. Diagrammatic sketch of continuously stirred tank reactor [47].

Table 1. The Parameters of CSTR model.

$C_A$	Concentration of species A	$\text{mol}\cdot\text{m}^{-3}$
$C_B$	Concentration of species B	$\text{mol}\cdot\text{m}^{-3}$
$C_C$	Concentration of species C	$\text{mol}\cdot\text{m}^{-3}$
$C_{AF}$	Feed concentration of species A	$\text{mol}\cdot\text{m}^{-3}$
$N_{BF}$	Molar feed rate of species B	$\text{mol}\cdot\text{s}^{-1}$
$F$	Volumetric feed rate	$\text{m}^3\cdot\text{s}^{-1}$

Define variables  $x_1 = \bar{x}_1 - x_{1s}$ ,  $x_2 = \bar{x}_2 - x_{2s}$ ,  $u = \bar{u} - u_s$ ,  $x_3 = \bar{x}_3 - x_{3s}$ ,  $z = \bar{z} - z_s$  where  $x_{1s}$ ,  $x_{2s}$ ,  $x_{3s}$ ,  $u_s$ ,  $z_s$  indicate the steady state values. To facilitate the analysis of the problem, we transform the tracking problem into a stabilization problem and discretize the CSTR system (16) with a sample period  $dT$ . The discrete-time model is:

$$\begin{cases} x_1(k+1) = (1 - dT - dT \times Da_1)x_1(k) \\ \quad + 2dT \times Da_2 \times x_{2s}x_2(k) \\ \quad + dT \times Da_2x_2(k)^2, \\ x_2(k+1) = dT \times Da_1x_1(k) \\ \quad - dT \times (Da_2 + Da_3)x_2(k)^2 \\ \quad + dT \times u + (1 - dT \\ \quad \times (1 + (2Da_2 + 2Da_3) \times x_{2s}))x_2(k), \\ x_3(k+1) = (1 - dT)x_3(k) + dT \times 2Da_3 \\ \quad \times x_{2s}x_2(k) + dT \times Da_3x_2(k)^2, \\ z = x_3(k). \end{cases} \quad (17)$$

For a CSTR system with intermittent faults, disturbance, parameter uncertainty, and time-delay, the control objective is to make the desired  $C_C$  as close as possible to its steady state value by adjusting  $N_{BF}$ . For system (1), giving sample period  $dT$ , one has:

$$x(k) = [x_1^T(k), x_2^T(k), x_3^T(k)]^T,$$

$$f = dT \times \begin{bmatrix} Da_2x_2(k)^2 \\ -(Da_2 + Da_3)x_2(k)^2 \\ Da_3x_2(k)^2 \end{bmatrix},$$

$$A = \begin{bmatrix} \theta_A & 2dT \times Da_2 \times x_{2s} & 0 \\ dT \times Da_1 & \sigma_A & 0 \\ 0 & 2dT \times Da_3 \times x_{2s} & 1 - dT \end{bmatrix}, \quad (18)$$

$$\theta_A = 1 - dT - dT \times Da_1,$$

$$\sigma_A = 1 - dT \times (1 + 2Da_2 \times x_{2s} + 2Da_3 \times x_{2s}).$$

The CSTR parameters are the same as [48]:  $Da_1 = 3$ ,  $Da_2 = 0.5$ ,  $Da_3 = 1$ ,  $x_{1s} = 0.3467$ ,  $x_{2s} = 0.8796$ ,  $x_{3s} = 0.8796$ . Other correlative parameters are chosen as follows:

$$A_d = F_d = D_1 = dT \times 0.1I, B_1 = dT \times [0 \ 1 \ 0]^T,$$

$$C = I, C_d = D_2 = 0.1I, M = [0.1 \ 0.1 \ 0.1]^T,$$

$$N_1 = N_2 = N_3 = N_4 = [0.1 \ 0.1 \ 0.1],$$

$$\Lambda(k) = 0.6 \sin(k), F = dT \times I, E = [0 \ 0 \ 1],$$

$$Y_0 = \begin{bmatrix} 3 & 0 & 1.5 \\ 0 & 3 & 0 \\ 1.5 & 0 & 3 \end{bmatrix}, Y_1 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 3 \end{bmatrix}, \quad (19)$$

$$Y_2 = \begin{bmatrix} 1 & 0.5 & 1 \\ 0.5 & 5 & 0 \\ 1 & 0 & 3 \end{bmatrix}, Y_3 = \begin{bmatrix} 1.5 & 0 & 1.5 \\ 0 & 3 & 0 \\ 1.5 & 0 & 4 \end{bmatrix}$$

$$w(k) = \begin{bmatrix} 0.5 \exp(-0.01k) \sin(0.01\pi k) \\ 0.5 \exp(-0.02k) \sin(0.02\pi k) \\ 0.5 \exp(-0.03k) \sin(0.03\pi k) \end{bmatrix}.$$

Let us suppose the sample period  $dT = 0.01$ , the nonlinearities are bounded by:

$$U_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, U_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (20)$$

$$V_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, V_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We assume the CSTR contains four modes and three of them indicate intermittent faults in actuator and sensors. Mode 1 is  $M_A(k) = 1$ ,  $M_S(k) = \text{diag}\{1, 1, 1\}$ . Mode 2 is  $M_A(k) = 1$ ,  $M_S(k) = \text{diag}\{0.8, 1, 0\}$ . Mode 3 is  $M_A(k) = 0.8$ ,  $M_S(k) = \text{diag}\{0, 0.8, 1\}$ . Mode 4 is  $M_A(k) = 0.8$ ,  $M_S(k) = \text{diag}\{1, 0, 0.8\}$ . When the CSTR switches among these four modes, we design fault-tolerant controller to guarantee a reliable and acceptable performance. The partly known transition probability matrix is given as follows:

$$P = \begin{bmatrix} 0.9 & ? & ? & ? \\ ? & 0.5 & ? & ? \\ ? & ? & 0.1 & ? \\ ? & ? & ? & 0.1 \end{bmatrix}.$$

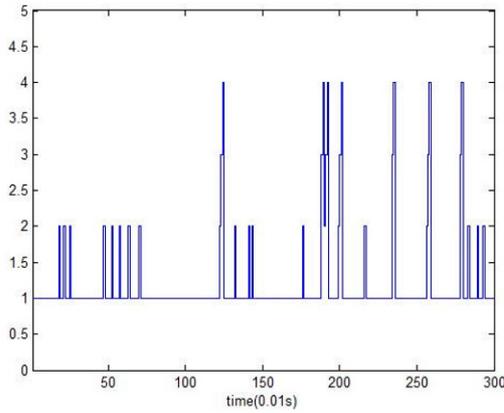


Fig. 5. Occurrence of intermittent faults in CSTR.

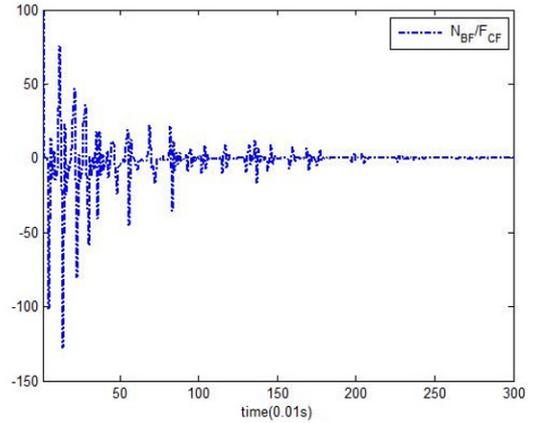


Fig. 8. The control signal of CSTR.

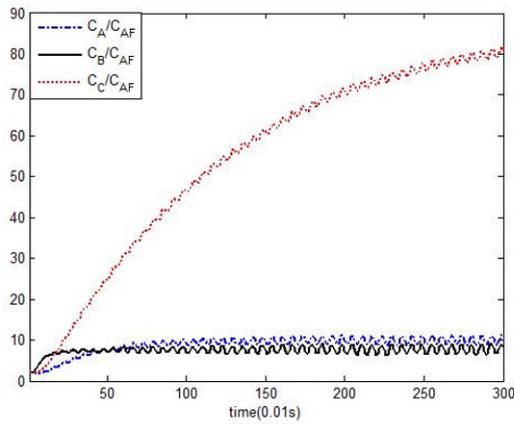


Fig. 6. State response without controller.

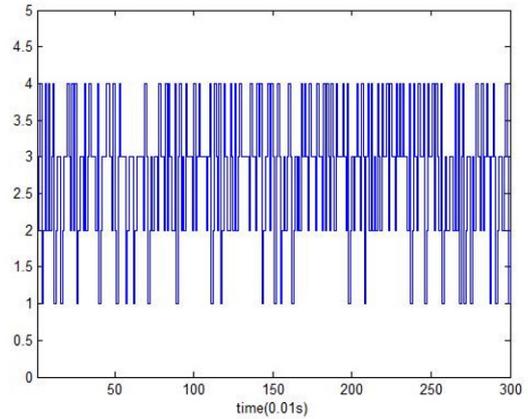


Fig. 9. Occurrence of intermittent faults in CSTR.

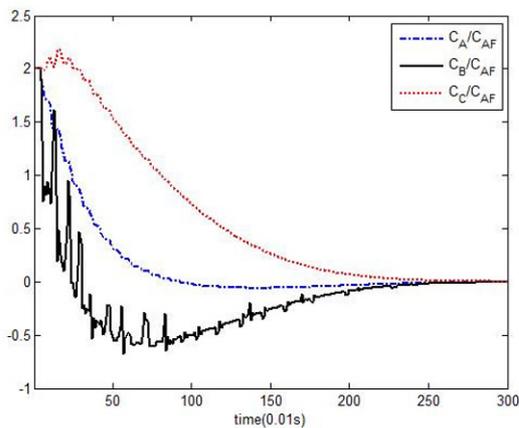


Fig. 7. State response of the controlled CSTR.

When substituting (18), (19), and (20) into (13) and (14), and solving the matrix inequalities (13) and (14), we can get the controller parameters. The simulation result can be seen in Figs. 5-12.

#### 4.1. A few intermittent faults

Fig. 5 shows that a series of IFs have occurred in CSTR. In Fig. 6, one can see the CSTR system is affected tremendously by IFs. Most notably, the desired  $C_C(x_3(k))$  is unstable. Fig. 7 shows the system becomes stable quickly and has favorable robust performance against disturbance and mode uncertainty. Fig. 8 shows the gradually convergent control input signals.

#### 4.2. Many intermittent faults

Fig. 9 shows the CSTR is subjected to more IFs than case 1 in sensors and actuator, the CSTR also works steadily, the effectiveness of the proposed fault-tolerant method is further validated. However, the CSTR state response without the controller diverges at a faster rate. From Figs. 10, 11, and 12, we can find that the size of the state response and controlled input of the controlled CSTR are bigger than case 1. It means that, if IFs occur with greater frequency, the performance of the system is affected more severely.

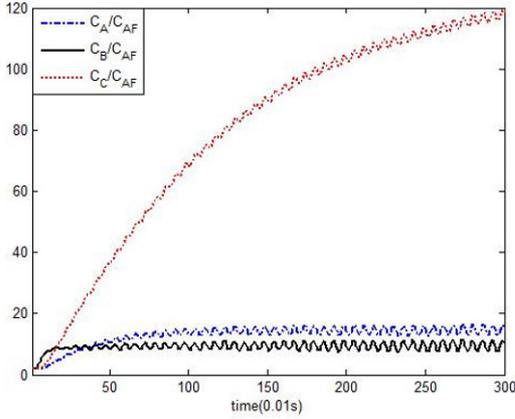


Fig. 10. State response without controller.

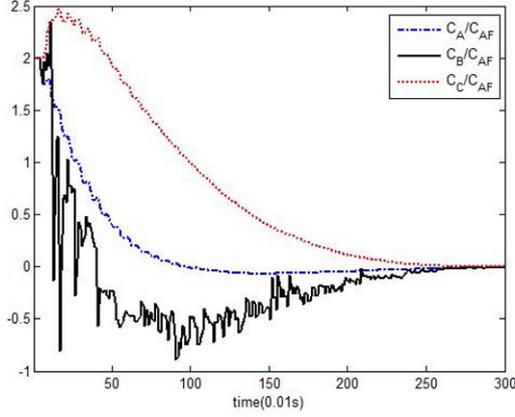


Fig. 11. State response of the controlled CSTR.

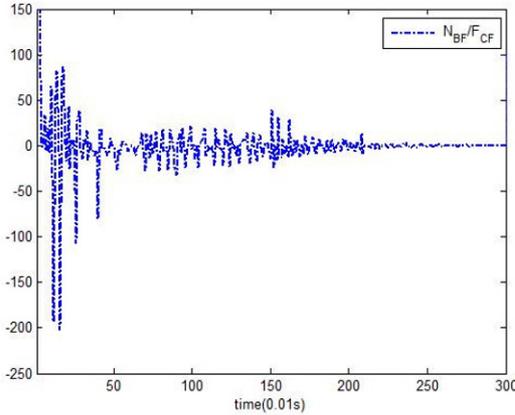


Fig. 12. The control signal of CSTR.

## 5. CONCLUSION

This study dealt with the FTC problem of IFs for non-linear uncertain systems with time-varying delays. According to the properties of IFs, the system subjected to IFs was transformed into a augmented MJS. Meanwhile, the FTC problem of IFs could be solved by using the sta-

bility analysis of a MJS. Moreover, the idea of robust control was used to reduce the effect of mode uncertainty and disturbance. The output feedback controller was obtained by using  $H_\infty$  control theory and the LMI approach. Finally, the validity of the developed method was demonstrated on CSTR. Based on this study, in the future, we will focus on the fault diagnosis and finite-time control problem of intermittent fault.

## APPENDIX A

In this part, we give the proof of Theorem 1.

### A.1. Stability of analysis

Define a Lyapunov-Krasovskii function as follows:

$$V(k, i) = V_1(k, i) + V_2(k) + V_3(k), \quad (\text{A.1})$$

where

$$\begin{aligned} V_1(k, i) &= \eta^T(k) P_i \eta(k), \\ V_2(k) &= \sum_{l=k-\tau(k)}^{k-1} \eta^T(l) T^T Q T \eta(l), \\ V_3(k) &= \sum_{\theta=k-\bar{\tau}+1}^{k-\tau} \sum_{l=\theta}^{k-1} \eta^T(l) T^T Q T \eta(l). \end{aligned} \quad (\text{A.2})$$

By labelling the mode at the  $k$ -th and  $(k+1)$ -th samples as  $i$  and  $j$ , respectively, under the condition  $w(k) = 0$ .  $P_i$  and  $Q$  are matrices that need to be determined. To deal with the problem of partly known transition probabilities, we define:

$$\begin{aligned} G_i &= \sum_{j \in \Psi_k^i} p_{ij} P_j + \sum_{j \in \Psi_U^i} p_{ij} P_j = P_K^i + \sum_{j \in \Psi_U^i} p_{ij} P_j, \\ P_i &= \sum_{j \in \Psi_K^i} p_{ij} P_i + \sum_{j \in \Psi_U^i} p_{ij} P_i = p_K^i P_i + \sum_{j \in \Psi_U^i} p_{ij} P_i. \end{aligned}$$

The difference of the Lyapunov-Krasovskii function can be obtained as follows:

$$\begin{aligned} E(\Delta V_1(k, i)) &= E(\zeta^T(k+1) G_i \zeta(k+1) \\ &\quad - \zeta^T(k) P_i \zeta(k)), \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} E(\Delta V_2(k)) &= E \left( \begin{aligned} &\zeta^T(k) T^T Q T \zeta(k) \\ &+ \sum_{l=k-\bar{\tau}+1}^{k-\tau} \zeta^T(l) T^T Q T \zeta(l) \\ &- \zeta^T(k-\tau(k)) T^T Q T \zeta(k-\tau(k)) \end{aligned} \right), \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} E(\Delta V_3(k)) &= E \left( \begin{aligned} &(\bar{\tau} - \tau) \zeta^T(k) T^T Q T \zeta(k) \\ &- \sum_{l=k-\bar{\tau}+1}^{k-\tau} \zeta^T(l) T^T Q T \zeta(l) \end{aligned} \right). \end{aligned} \quad (\text{A.5})$$

Notice that the variable in the difference of the Lyapunov-Krasovskii function and define:

$$\eta^T(k) = \begin{bmatrix} \zeta^T(k) & x^T((k-\tau(k))) & f^T(k) & f_d^T((k-\tau(k))) \end{bmatrix}.$$

Then, sum up the left and right side of the difference of the Lyapunov-Krasovskii function, one can get:

$$E(\Delta V(k, i)) \leq \eta^T(k) \Theta_i \eta(k), \quad (\text{A.6})$$

where

$$\Theta_i = \begin{bmatrix} \Pi_1 & \bar{A}_i^T P_K^j \bar{A}_{di} & \bar{A}_i^T P_K^j \bar{F}_i & \bar{A}_i^T P_K^j \bar{F}_{di} \\ * & \bar{A}_{di}^T P_K^j \bar{A}_{di} - p_K^j Q & \bar{A}_{di}^T P_K^j \bar{F}_i & \bar{A}_{di}^T P_K^j \bar{F}_{di} \\ * & * & \bar{F}_i^T P_K^j \bar{F} & \bar{F}_i^T P_K^j \bar{F}_{di} \\ * & * & * & \bar{F}_{di}^T P_K^j \bar{F}_{di} \end{bmatrix} + \sum_{j \in \Psi_b^i} p_{ij} \begin{bmatrix} \bar{\Pi}_1 & \bar{A}_i^T P_j \bar{A}_{di} & \bar{A}_i^T P_j \bar{F}_i & \bar{A}_i^T P_j \bar{F}_{di} \\ * & \bar{A}_{di}^T P_j \bar{A}_{di} - Q & \bar{A}_{di}^T P_j \bar{F}_i & \bar{A}_{di}^T P_j \bar{F}_{di} \\ * & * & \bar{F}_i^T P_j \bar{F} & \bar{F}_i^T P_j \bar{F}_{di} \\ * & * & * & \bar{F}_{di}^T P_j \bar{F}_{di} \end{bmatrix},$$

$$\Pi_1 = \bar{A}_i^T P_K^j \bar{A}_i - p_K^j P_i + p_K^j (1 + \bar{\tau} - \underline{\tau}) T^T Q T,$$

$$\bar{\Pi}_1 = \bar{A}_i^T P_j \bar{A}_i - P_i + (1 + \bar{\tau} - \underline{\tau}) T^T Q T.$$

Notice that sector-bounded conditions (3) and (4), which implies

$$\begin{aligned} [f(x(k)) - U_1(x(k))]^T [f(x(k)) - U_2(x(k))] &\leq 0, \\ [f_d(x(k)) - V_1(x(k))]^T [f_d(x(k)) - V_2(x(k))] &\leq 0, \end{aligned}$$

or equivalently,

$$\begin{aligned} &\mu \begin{bmatrix} \zeta^T(k) & f^T(x(k)) \end{bmatrix} \\ &\times \begin{bmatrix} T^T \bar{U}_1 T & T^T \bar{U}_2 \\ * & I \end{bmatrix} \begin{bmatrix} \zeta(k) \\ f(x(k)) \end{bmatrix} \leq 0, \\ &\lambda \begin{bmatrix} x^T(k - \tau(k)) & f_d^T(x(k - \tau(k))) \end{bmatrix} \\ &\times \begin{bmatrix} \bar{V}_1 & \bar{V}_2 \\ * & I \end{bmatrix} \begin{bmatrix} x(k - \tau(k)) \\ f_d(x(k - \tau(k))) \end{bmatrix} \leq 0, \end{aligned} \quad (\text{A.7})$$

where  $\bar{U}_1 = (U_1^T U_2 + U_2^T U_1)/2$ ,  $\bar{U}_2 = -(U_1^T + U_2^T)/2$ ,  $\bar{V}_1 = (V_1^T V_2 + V_2^T V_1)/2$ ,  $\bar{V}_2 = -(V_1^T + V_2^T)/2$ ,  $\mu \geq 0$ ,  $\lambda \geq 0$ . According to Lemma 1 and Lemma 3, it can be inferred that the right side of (A.6) is negative, one gets:

$$E(\Delta V(k, i)) < 0. \quad (\text{A.8})$$

Hence, one can conclude that:

$$E(\Delta V(k, i)) \leq \lambda_{\min}(\Theta_i) \eta^T(k) \eta(k) \leq -\alpha \|\eta(k)\|^2, \quad (\text{A.9})$$

where  $\lambda_{\min}(\Theta_i)$  signifies the minimum eigenvalue of  $\Theta_i$ , and  $\alpha = \inf\{\lambda_{\min}(-\Theta_i)\}$ . Summing (A.9) on both sides from 0 to  $K$ , one has:

$$E\left(\sum_{k=0}^K \|\eta^2(k)\|\right)$$

$$\begin{aligned} &\leq \frac{1}{\alpha} E(V(\zeta(0), r(0)) - V(\zeta(K), r(K))) \\ &\leq \frac{1}{\alpha} E(V(\zeta(0), r(0))) < \infty. \end{aligned} \quad (\text{A.10})$$

Further, it can be directly obtained that:

$$\sum_{k=0}^{\infty} E(\zeta^T(k) \zeta(k) | \zeta(0), r(0)) < \infty. \quad (\text{A.11})$$

Therefore, according to Definition 1, we can conclude that the uncertain nonlinear system with time-varying delays and IFs is stochastically stable.

## A.2. performance analysis

Next, based on Appendix A.1, we focus on the analysis of the  $H_\infty$  performance of resulted system (7). In order to evaluate the  $H_\infty$  performance, the following index is introduced:

$$J = E\left\{\sum_{k=0}^{\infty} [z^T(k) z(k) - \gamma^2 w^T(k) w(k)]\right\}. \quad (\text{A.12})$$

Under the zero initial condition, it is not difficult to verify that

$$\begin{aligned} J &= E\left(\sum_{k=0}^{\infty} (z^T(k) z(k) - \gamma^2 w^T(k) w(k))\right) \\ &= E\left(\sum_{k=0}^{\infty} \left(z^T(k) z(k) - \gamma^2 w^T(k) w(k) + \Delta V(k, i)\right)\right) \\ &\quad + E\{V(\zeta(0), r(0))\} - E\{V(\zeta(\infty), r(\infty))\} \\ &\leq E\left(\sum_{k=0}^{\infty} \left(z^T(k) z(k) - \gamma^2 w^T(k) w(k) + \Delta V(k, i)\right)\right). \end{aligned} \quad (\text{A.13})$$

Our goal is to prove that  $J < 0$ . Combining (7), (A.1), and (A.13), it can be seen that for any nonzero  $w(k) \in l_2[0, \infty)$

$$\begin{aligned} J &\leq E\{z^T(k) z(k) - \gamma^2 w^T(k) w(k) + \Delta V(k, i)\} \\ &\leq E(\phi^T(k) \bar{\Theta} \phi(k)), \end{aligned} \quad (\text{A.14})$$

where

$$\begin{aligned} \phi^T(k) &= \begin{bmatrix} \zeta^T(k) & x^T((k-\tau(k))) & f^T(k) \\ & f_d^T((k-\tau(k))) & w^T(k) \end{bmatrix}, \\ \bar{\Theta} &= \begin{bmatrix} \Pi_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & -Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \bar{A}_i^T & E_i^T \\ \bar{A}_{di}^T & \mathbf{0} \\ \bar{F}_i^T & \mathbf{0} \\ \bar{F}_{di}^T & \mathbf{0} \\ D_i^T & \mathbf{0} \end{bmatrix} \\ &\quad \times \begin{bmatrix} P_K^j + \sum_{j \in \Psi_b^i} p_{ij} P_j & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix} \begin{bmatrix} \bar{A}_i & \bar{A}_{di} & \bar{F}_i & \bar{F}_{di} & D_i \\ E_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \end{aligned}$$

$$\Pi_2 = (1 + \bar{\tau} - \underline{\tau}) T^T Q T - p_K^j P_i - \sum_{j \in \Psi_b^i} p_{ij} P_j.$$

In order to prove  $J < 0$ , we need to prove that  $\bar{\Theta} < 0$ . Therefore, applying the Schur complement to (11) and

(12), one has:

$$\begin{aligned} & \begin{bmatrix} \bar{\Pi}_2 & \mathbf{0} & -\mu T^T \bar{U}_2 & \mathbf{0} & \mathbf{0} \\ * & -Q - \lambda V_1 & \mathbf{0} & -\lambda \bar{V}_2 & \mathbf{0} \\ * & * & -\mu I & \mathbf{0} & \mathbf{0} \\ * & * & * & -\lambda I & \mathbf{0} \\ * & * & * & * & -\gamma^2 I \end{bmatrix} \\ & + \begin{bmatrix} \bar{A}_i^T & E_i^T \\ \bar{A}_{di}^T & \mathbf{0} \\ \bar{F}_i^T & \mathbf{0} \\ \bar{F}_{di}^T & \mathbf{0} \\ D_i^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} P_K^j + \sum_{j \in \Psi_i} p_{ij} P_j & \mathbf{0} \\ \mathbf{0} & I \end{bmatrix} \\ & \times \begin{bmatrix} \bar{A}_i & \bar{A}_{di} & \bar{F}_i & \bar{F}_{di} & D_i \\ E_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0. \end{aligned}$$

The rest of the proof is similar to the proof of stochastic stability, according to Lemma 3, one knows:

$$J \leq E(\phi^T(k) \bar{\Theta} \phi(k)) < 0. \quad (\text{A.15})$$

It means that  $\|z(k)\|_2^2 - \gamma^2 \|w(k)\|_2^2 < 0$ . Hence, resulted system (7) is stochastic stability with given  $H_\infty$  performance index. This completes the proof.

## APPENDIX B

In this part, we give the proof of Theorem 2.

### B.1. Parameter uncertainties analysis

First, we deal with the problem of parameter uncertainties. In Theorem 1, according to the definition of time-varying parameter uncertainties, combining (2) and (7) with matrix inequalities (11) and (12), one can get:

$$\begin{aligned} \Upsilon_i &= \Upsilon_{iK} + \Upsilon_{iU} \\ &= \Xi_i + \bar{N}^T \Lambda^T(k) \bar{M}^T + \bar{M} \Lambda(k) \bar{N} < 0, \end{aligned} \quad (\text{B.1})$$

where

$$\begin{aligned} \bar{N} &= [N_1 \quad N_2 \quad N_3 \quad N_4 \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}], \\ \bar{M}^T &= [\mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0} \quad M^T \quad \mathbf{0}], \\ \Xi_i &= \begin{bmatrix} \bar{\Pi}_2 & \mathbf{0} & -\mu T^T \bar{U}_2 & \mathbf{0} & \mathbf{0} & A_i^T & E_i^T \\ * & \chi_\Xi & \mathbf{0} & -\lambda \bar{V}_2 & \mathbf{0} & A_{di}^T & \mathbf{0} \\ * & * & -\mu I & \mathbf{0} & \mathbf{0} & F_i^T & \mathbf{0} \\ * & * & * & -\lambda I & \mathbf{0} & F_{di}^T & \mathbf{0} \\ * & * & * & * & -\gamma^2 I & D_i^T & \mathbf{0} \\ * & * & * & * & * & P_{pi} & \mathbf{0} \\ * & * & * & * & * & * & -I \end{bmatrix}, \\ P_{pi} &= - \left( \sum_{j \in \Psi_k} p_{ij} P_j + \sum_{j \in \Psi_i} p_{ij} P_j \right)^{-1}, \quad \chi_\Xi = -Q - \lambda \bar{V}_1. \end{aligned}$$

According to Lemma 2, for matrices  $\Xi_i, \bar{M}, F(k), \bar{N}$  with appropriate dimensions,  $F(k)^T F(k) \leq I$ , and any scalar  $\varepsilon > 0$ , it can be obtained that:

$$\begin{aligned} & \Xi_i + \bar{N}^T F^T(k) \bar{M}^T + \bar{M} F(k) \bar{N} \\ & \leq \Xi_i + \varepsilon^{-1} \bar{M} \bar{M}^T + \varepsilon \bar{N}^T \bar{N} \doteq \bar{\Xi}_i. \end{aligned} \quad (\text{B.2})$$

Obviously, if  $\bar{\Xi}_i < 0$ , (11) and (12) hold.

### B.2. Controller analysis

Next, our goal is to obtain the output feedback controller in the form of (5) such that the closed-loop system in (7) is asymptotically stable with an  $H_\infty$  disturbance attention level. Supposing  $\bar{\Xi}_i < 0$ , we can get concrete controller parameters and the LMI condition. Define the following partitioned matrices:

$$\begin{aligned} P_i &= \begin{bmatrix} X_i & R_i \\ * & \bar{X}_i \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} Y_i & Y_i \\ * & \bar{Y}_i \end{bmatrix}, \\ \Xi_1 &= \begin{bmatrix} Y_i & I \\ Y_i^T & \mathbf{0} \end{bmatrix}, \quad G_i = \sum_{j \in \Psi} p_{ij} P_j = \begin{bmatrix} \hat{X}_i & \hat{R}_i \\ * & \hat{X}_i \end{bmatrix}, \\ G_i^{-1} &= \begin{bmatrix} H_{1i} & H_{2i} \\ * & H_{3i} \end{bmatrix}, \quad \Xi_2 = \begin{bmatrix} I & \hat{X}_i \\ \mathbf{0} & \hat{R}_i \end{bmatrix}, \\ \hat{R}_i &= \hat{Y}_{qi}^{-1} - \hat{X}_i. \end{aligned} \quad (\text{B.3})$$

By using the congruence transformation  $\text{diag}\{\Xi_1^T, I, I, I, I, \Xi_2^T, I, I, I\}$ , we get some block multiplications:

$$\begin{aligned} \Phi_{1i}^T &= Y_i C_{ci}^T, \quad \Phi_{3i}^T = B_{ci}^T \hat{R}_i^T, \\ \Phi_{2i}^T &= Y_i A^T \hat{X}_i + \Phi_{1i}^T M_A(k)^T B_1^T \hat{X}_i \\ & \quad + Y_i C^T M_S(k)^T \Phi_{3i}^T + Y_i A_{ci} \hat{R}_i. \end{aligned} \quad (\text{B.4})$$

From (B.4), we can conclude that the output feedback controller parameters can be calculated in (15). Additionally, according to [30] and referring to its Theorem 1, we have:

$$H_{1i} \leq \hat{Y}_{qi} + \hat{Y}_{qi}^T - \hat{Y}_{qi}. \quad (\text{B.5})$$

According to block multiplications and (B.5), controller parameters (15), LMI conditions (13) and (14) can easily be obtained. This completes the proof.

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