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Fault estimation and compensation for two-dimensional linear systems with actuator/sensor faults

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Abstract: This paper focuses on the problem of actuator/sensor reconstruction and compensation for two-dimensional linear systems. For a two-dimensional system described by the Fornasini-Marchesini second model, the simultaneous state and faults estimation scheme is developed, and a sufficient condition is presented and proved for the asymptotic stability of the estimator. Based on the estimation of actuator/sensor faults, the faults compensation proposal can be performed by subtracting the actuator faults and sensor faults. Finally, some simulation results are presented to demonstrate the performance of faults estimation and fault compensation.

Keywords: Two-dimensional systems, simultaneous estimation, observer, actuator/sensor faults, fault compensation

1. Introduction

With the rapid development of modern society, people need to process multi-dimensional signals in multi-dimensional systems[1]. Based on this situation, a series of multi-dimensional models applied to the actual situation have received extensive attention in academia[2, 3]. The two-dimensional system, for its representativeness, has been devoted a great deal of effort and many important conclusions and applications[4]. It's well-known that, the studies of two-dimensional (2-D) systems can be traced back to the discussions that Ansell introduced in the analysis of power grids in the 1960s. More and more 2-D system models based on different state-space description have been brought afterward, such as Roesser model for image data processing and multidimensional linear iterative circuits[5]. And batch process analysis[6], digital picture processing[7] take advantage of Roesser model. In this paper, we discuss the more general situation, FM-II model, because the Rosser model and the FM-I model are both special cases of FM-II model[8, 9].

Some results about filtering[10], 2-D observers and state estimation, fault detection and isolation have been reflected in literatures. The estimation problems of 2-D systems have received much attention in the past decades. Various kinds of 2-D system observers have been designed, such as nonlinear system observers[11], singular system observers[12], unknown input observers (UIO)[13], and so on. In [14] , Cao et al have designed an observer can estimate state and multiple faults simultaneously for 2-D system with multiple faults.

As we know, there are numbers of publications on the estimation of state and sensor fault [15, 16]. However, in the practical situation, for 2-D systems, the fault in state equation and in measurement equation can occur simultaneously. At this time, fault reconstruction for the 2-D systems with actuator/process faults is important to ensure system performance[17]. Gao and Ding have done some pioneering work for 1-D systems[18, 19]. In this paper, by using the observer scheme in [14], simultaneous estimation of state and multiple faults has been designed for 2-D systems and the fault reconstruction and compensation.

To the authors' best knowledge, this study has the following contributions: first, we use the conclusions to fault compensation in 2-D linear system with actuator/sensor faults for the first time. Based on the result of simultaneous

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estimation of the actuator/sensor faults, a simple and effective faults reconstruction and compensation scheme will be put forward. Second, without the switchover of actuator and sensor, the output feedback controller can operate normally.

The rest of this paper is organized as follows. In part 2, the 2-D FM-II model is given, and preliminary knowledge is presented. In part 3, an asymptotic stable observer is given to estimate actuator/sensor faults. The part 4 introduces the faults compensation. In part 5, an example is given to demonstrate the effectiveness of the proposed scheme. Part 6 summarizes the results of this study.

Notations. In this paper, $X \in R^{m\times n}$ indicates that X is a $m\times n$ real matrix. $X>0$ means that the real matrix X is positive definite. The symbol $\|*\|$ means 2-norm. $X^{\dagger}$ means the Moore-Penrose pseudo-inverse of matrix X.

2. Problem formulation and preliminary

Consider a 2-D system described by the FM-II model with actuator/sensor faults as follows:

$$\begin{cases} x(i+1,j+1) = A_1x(i,j+1) + A_2x(i+1,j) + B_1u(i,j+1) \\ \qquad + B_2u(i+1,j) + M_1f_a(i,j+1) + M_2f_a(i+1,j) \\ y(i,j) = Cx(i,j) + Du(i,j) + f_s(i,j) \end{cases} \quad (1)$$

and the unknown boundary conditions satisfy:

$$\begin{aligned} &\sup\|x(i,0)\| < \infty, i=0,1,...; \sup\|x(0,j)\| < \infty, j=1,2,... \\ &\sup\|f_a(i,0)\| < \infty, i=0,1,...; \sup\|f_a(0,j)\| < \infty, j=1,2,... \\ &\sup\|f_s(i,0)\| < \infty, i=0,1,...; \sup\|f_s(0,j)\| < \infty, j=1,2,... \end{aligned} \quad (2)$$

where $x(i,j) \in R^n, y(i,j) \in R^p$, and $u(i,j) \in R^m$ are the system state vector, output measurement vector and input vector; $f_a(i,j) \in R^q$ is the actuator fault or process fault in the state equation, $f_s(i,j) \in R^p$ is the sensor fault in the measurement equation; $A_k, B_k, M_k (k=1,2), C, D$ are system matrices with appropriate dimensions.

From the fault description in 1-D systems[20], Cao et al give the dynamic characteristics of $f_a(i,j) \in R^q$ for 2-D systems as follows[14]:

$$f_a(i+1,j+1) = A_{d1}f(i,j+1) + A_{d2}f(i+1,j) \quad (3)$$

where A_{d1}, A_{d2} are known matrices with appropriate dimensions.

Lemma 1 [21]. For any matrices or vectors w, h and any positive definite matrix X with appropriate dimensions, the following inequality always holds

$$hw + w^Th^T \le hXh^T + w^TX^{-1}w. \quad (4)$$

Lemma 2 [10, 22, 23]. The 2-D system (1) is asymptotically stable if there is a positive definite and radially unbounded function $V(i,j): R^n \to R$ and $\alpha, \beta > 0$ *and* $\alpha + \beta = 1$, such that

$$\begin{aligned} &\Delta V(i,j) = \\ &\quad V(x(i+1,j+1)) - \alpha V(x(i,j+1)) - \beta V(x(i+1,j)) < 0. \end{aligned} \quad (5)$$

for all $\begin{bmatrix} x^T(i,j+1) & x^T(i+1,j) \end{bmatrix}^T \ne 0$

3. Simultaneous actuator/sensor faults estimation

In this part, we will propose an observer design scheme to make observer asymptotic stable for system (1) with or without input disturbance.

Define

$$\begin{aligned} &E = \begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \bar{A}_1 = \begin{bmatrix} A_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} A_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} B_1 \\ \mathbf{0} \end{bmatrix}, \\ &\bar{B}_2 = \begin{bmatrix} B_2 \\ \mathbf{0} \end{bmatrix}, \bar{M}_1 = \begin{bmatrix} M_1 \\ 0 \end{bmatrix}, \bar{M}_2 = \begin{bmatrix} M_2 \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & I_p \end{bmatrix}, \\ &\bar{x}(i+1,j+1) = \begin{bmatrix} x^T(i+1,j+1) & f_s^T(i+1,j+1) \end{bmatrix}^T. \end{aligned} \quad (6)$$

According to the 2-D system (1), one can obtain the augmented 2-D singular system as follows:

$$\begin{cases} E\bar{x}(i+1,j+1) = \bar{A}_1\bar{x}(i,j+1) + \bar{A}_2\bar{x}(i+1,j) + \bar{B}_1u(i,j+1) \\ \qquad + \bar{B}_2u(i+1,j) + \bar{M}_1f_a(i,j+1) + \bar{M}_2f_a(i+1,j) \\ y(i,j) = \bar{C}\bar{x}(i,j) + Du(i,j) \end{cases} \quad (7)$$

Consider the observer as follows:

$$\begin{cases} z(i+1,j+1) = F_1z(i,j+1) + F_2z(i+1,j) + G_1u(i,j+1) \\ \qquad + G_2u(i+1,j) + H_1y(i,j+1) + H_2y(i+1,j) \\ \qquad + S_1\hat{f}_a(i,j+1) + S_2\hat{f}_a(i+1,j) \\ \hat{f}_a(i+1,j+1) = A_{d1}\hat{f}_a(i,j+1) + A_{d2}\hat{f}_a(i+1,j) \\ \qquad - K_{d1}\bar{C}z(i,j+1) - K_{d2}\bar{C}z(i+1,j) \\ \hat{\bar{x}}(i,j) = z(i,j) + Ty(i,j) \end{cases} \quad (8)$$

Theorem 1. For 2-D system (7) and following observer matrices:

$$\begin{aligned} &L, K_1, K_2 \in R^{(n+p)\times p}, \ K_{d1}, K_{d1} \in R^{q\times p}, \\ &F_1 = (E + L\bar{C})^{-1}(\bar{A}_1 - K_1\bar{C}), F_2 = (E + L\bar{C})^{-1}(\bar{A}_2 - K_2\bar{C}), \\ &G_1 = (E + L\bar{C})^{-1}\bar{B}_1, G_2 = (E + L\bar{C})^{-1}\bar{B}_2, \\ &S_1 = (E + L\bar{C})^{-1}\bar{M}_1, S_2 = (E + L\bar{C})^{-1}\bar{M}_2, \\ &H_1 = (E + L\bar{C})^{-1}\bar{A}_1(E + L\bar{C})^{-1}L, \\ &H_2 = (E + L\bar{C})^{-1}\bar{A}_2(E + L\bar{C})^{-1}L, T = (E + LC)^{-1}L \end{aligned} \quad (9)$$

If there exist a positive definite matrix $P \in R^{(n+p+q)\times(n+p+q)}$, matrices Z_1, Z_2, and positive scalars $\alpha, \beta, (\alpha+\beta=1)$ such that linear matrix inequality as follows holds:

$$\begin{bmatrix} -\alpha P & * & * \\ 0 & -\beta P & * \\ P(\tilde{E}_L)^{-1}\tilde{A}_1 - Z_1\tilde{C} & P(\tilde{E}_L)^{-1}\tilde{A}_2 - Z_2\tilde{C} & -P \end{bmatrix} < 0 \quad (10)$$

$$J_1 = (P\tilde{E}_L^{-1})^{-1}Z_1, J_2 = (P\tilde{E}_L^{-1})^{-1}Z_2. \quad (11)$$

then $\lim_{i,j\to\infty}(\bar{x}(i,j)-\hat{\bar{x}}(i,j))=0$ and $\lim_{i,j\to\infty}(f(i,j)-\hat{f}(i,j))=0$, and system (8) is an asymptotic observer for system (1).

Proof. Partitioning $L=\begin{bmatrix}L_1^T & L_2^T\end{bmatrix}^T$, where $L_1\in R^{n\times p}, L_2\in R^{p\times p}$, when

$$\begin{aligned}&rank(E+L\bar{C})=rank(\begin{bmatrix}I_n & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}L_1\\ L_2\end{bmatrix}\begin{bmatrix}C & I_p\end{bmatrix})\\&=rank\begin{bmatrix}I_n & L_1\\ \mathbf{0} & L_2\end{bmatrix}=rank(L_2)+n\end{aligned}\tag{12}$$

It's clearly that if and only if the square matrix L_2 is of full rank, $(E+L\bar{C})$ is nonsingular. Therefore, letting L_2 be nonsingular, the relationship can be obtained as follows:

$$\begin{aligned}&(E+L\bar{C})^{-1}=\begin{bmatrix}I_n & -L_1(L_2)^{-1}\\ -C & (I_p+CL_1)(L_2)^{-1}\end{bmatrix},\\&\bar{C}(E+L\bar{C})^{-1}L=\\&\begin{bmatrix}C & I_p\end{bmatrix}\begin{bmatrix}I_n & -L_1(L_2)^{-1}\\ -C & (I_p+CL_1)(L_2)^{-1}\end{bmatrix}\begin{bmatrix}L_1\\ L_2\end{bmatrix}\\&=\begin{bmatrix}C & I_p\end{bmatrix}\begin{bmatrix}\mathbf{0}\\ I_p\end{bmatrix}=I_p.\end{aligned}\tag{13}$$

Adding $Ly(i+1,j+1)$ to both sides of the first equation in (7), it yields

$$\begin{aligned}&(E+L\bar{C})\bar{x}(i+1,j+1)=\bar{A}_1\bar{x}(i,j+1)+\bar{A}_2\bar{x}(i+1,j)\\&+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)+\bar{M}_1f_a(i,j+1)\\&+\bar{M}_2f_a(i+1,j)+Ly(i+1,j+1)\\&=(\bar{A}_1-K_1\bar{C})\bar{x}(i,j+1)+(\bar{A}_2-K_2\bar{C})\bar{x}(i+1,j)\\&+K_1y(i,j+1)+K_2y(i+1,j)+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)\\&+\bar{M}_1f_a(i,j+1)+\bar{M}_2f_a(i+1,j)+Ly(i+1,j+1).\\&f_a(i+1,j+1)=A_{d1}f_a(i,j+1)+A_{d2}f_a(i+1,j)\end{aligned}\tag{14}$$

Substituting $z(i,j)=\bar{x}(i,j)-(E+L\bar{C})^{-1}Ly(i,j)$ into observer shown in (8), using equation (9) and equation (13), the observer in (8) becomes:

$$\begin{cases}(E+L\bar{C})\hat{\bar{x}}(i+1,j+1)=\\(\bar{A}_1-K_1\bar{C})\hat{\bar{x}}(i,j+1)+(\bar{A}_2-K_2\bar{C})\hat{\bar{x}}(i+1,j)\\+\bar{B}_1u(i,j+1)+\bar{B}_2u(i+1,j)+K_1y(i,j+1)+K_2y(i+1,j)\\+\bar{M}_1\hat{f}_a(i,j+1)+\bar{M}_2\hat{f}_a(i+1,j)+Ly(i+1,j+1).\\\hat{f}_a(i+1,j+1)=A_{d1}\hat{f}_a(i,j+1)+A_{d2}\hat{f}_a(i+1,j)\\+K_{d1}\bar{C}[\bar{x}(i,j+1)-\hat{\bar{x}}(i,j+1)]+K_{d2}\bar{C}[\bar{x}(i+1,j)-\hat{\bar{x}}(i+1,j)].\end{cases}\tag{15}$$

Define $e(i,j)=\bar{x}(i,j)-\hat{\bar{x}}(i,j), e_d(i,j)=f_a(i,j)-\hat{f}_a(i,j).$ It can get the error dynamic system as follows:

$$\begin{aligned}&e_d(i+1,j+1)=A_{d1}e_d(i,j+1)+A_{d2}e_d(i+1,j)\\&-K_{d1}\bar{C}e(i,j+1)-K_{d2}\bar{C}e(i+1,j),\\&(E+L\bar{C})e(i+1,j+1)=(\bar{A}_1-K_1\bar{C})e(i,j+1)\\&+(\bar{A}_2-K_2\bar{C})e(i+1,j)+\bar{M}_1e_d(i,j+1)+\bar{M}_2e_d(i+1,j).\end{aligned}\tag{16}$$

Rewrite (16) into a compact form as follows:

$$\begin{aligned}&\underbrace{\begin{bmatrix}E+L\bar{C} & \mathbf{0}\\ \mathbf{0} & I_q\end{bmatrix}}_{\tilde{E}_L}\underbrace{\begin{bmatrix}e(i+1,j+1)\\ e_d(i+1,j+1)\end{bmatrix}}_{\tilde{e}(i+1,j+1)}=\\&\left(\underbrace{\begin{bmatrix}\bar{A}_1 & \bar{M}_1\\ \mathbf{0} & A_{d1}\end{bmatrix}}_{\tilde{A}_1}-\underbrace{\begin{bmatrix}K_1\\ K_{d1}\end{bmatrix}}_{J_1}\underbrace{\begin{bmatrix}\bar{C} & \mathbf{0}\end{bmatrix}}_{\tilde{C}}\right)\underbrace{\begin{bmatrix}e(i,j+1)\\ e_d(i,j+1)\end{bmatrix}}_{\tilde{e}(i,j+1)}\\&+\left(\underbrace{\begin{bmatrix}\bar{A}_2 & \bar{M}_2\\ \mathbf{0} & A_{d2}\end{bmatrix}}_{\tilde{A}_2}-\underbrace{\begin{bmatrix}K_2\\ K_{d2}\end{bmatrix}}_{J_2}\underbrace{\begin{bmatrix}\bar{C} & \mathbf{0}\end{bmatrix}}_{\tilde{C}}\right)\underbrace{\begin{bmatrix}e(i+1,j)\\ e_d(i+1,j)\end{bmatrix}}_{\tilde{e}(i+1,j)}\end{aligned}\tag{17}$$

Furthermore, it can be rewritten as:

$$\begin{aligned}&\tilde{e}(i+1,j+1)=\tilde{E}_L^{-1}(\tilde{A}_1-J_1\tilde{C})\tilde{e}(i,j+1)\\&+\tilde{E}_L^{-1}(\tilde{A}_2-J_2\tilde{C})\tilde{e}(i+1,j)=\begin{bmatrix}\tilde{F}_1 & \tilde{F}_2\end{bmatrix}\begin{bmatrix}\tilde{e}(i,j+1)\\ \tilde{e}(i+1,j)\end{bmatrix}\\&=\tilde{F}\tilde{e}.\end{aligned}\tag{18}$$

In accordance with **lemma 1**, let P be a positive definite symmetric matrix.

Choose $V=e(i+1,j+1)^TPe(i+1,j+1)$ as radially unbounded Lyapunov function. Then, it can get as follows:

$$\begin{aligned}&\Delta V=\tilde{e}(i+1,j+1)^TP\tilde{e}(i+1,j+1)\\&-\alpha\tilde{e}^T(i,j+1)P\tilde{e}(i,j+1)-\beta\tilde{e}^T(i+1,j)P\tilde{e}(i+1,j)\\&\alpha,\beta>0,\alpha+\beta=1\end{aligned}\tag{19}$$

From (18), it yields:

$$\Delta V=\tilde{e}^T(\tilde{F}^TP\tilde{F}-\begin{bmatrix}\alpha P & 0\\ 0 & \beta P\end{bmatrix})\tilde{e}\tag{20}$$

According to equation (10) and Schur complement, we can get:

$$\tilde{F}^TP\tilde{F}-\begin{bmatrix}\alpha P & 0\\ 0 & \beta P\end{bmatrix}<0\tag{21}$$

Referring to Lemma 1, error dynamic system (18) is asymptotically stable for 2-D system, and **Theorem 1** is proved.

4. Fault compensation

Consider the original FM-II model with sensor/actuator faults and noise, the stability of the closed-loop 2-D system cannot be guaranteed normally. With the simultaneous estimation of multiple channel faults, the actuator/sensor faults compensation can be performed well.

As $\hat{\bar{x}}(i+1,j+1)=\left[\hat{x}^T(i+1,j+1) \quad \hat{f}_s^T(i+1,j+1)\right]^T$, the estimation of sensor fault is

$$\hat{f}_s(i+1,j+1)=\left[0 \quad I_p\right]\hat{\bar{x}}(i+1,j+1)=\tau\hat{\bar{x}}(i+1,j+1). \quad (22)$$

For system (1), subtracting $\hat{f}_s(i,j)$ from the output $y(i,j)$ and for discussion that $D=0$, yields the compensated output.

$$\begin{aligned} y_c(i,j) &= y(i,j)-f_s(i,j)=Cx(i,j)+\tau\bar{x}(i,j)-\tau\hat{\bar{x}}(i,j) \\ &= Cx(i,j)+\tau e(i,j) \end{aligned} \quad (23)$$

Let

$$\begin{aligned} &B=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix},\ M=\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}, \\ &L_f=B^{\dagger}M. \end{aligned} \quad (24)$$

with $rank\left[B \quad M\right]=rankB$, then one gets

$$M-BL_f=0 \quad (25)$$

Subtracting $L_f\hat{f}_a$ from the actuator input, so the output feedback controller can be compensated without sensor fault and actuator fault as follows:

$$u=-K_F y_c-L_f\hat{f}_a. \quad (26)$$

Substituting (22) and (24) into the dynamical equation of the plant (1), one can get

$$\begin{aligned} &x(i+1,j+1)=(A_1-B_1K_FC)x(i,j+1) \\ &+(A_2-B_2K_FC)x(i+1,j)-B_1K_F\tau e(i,j+1) \\ &-B_2K_F\tau e(i+1,j)+M_1e_d(i,j+1)+M_2e_d(i+1,j) \end{aligned} \quad (27)$$

Define

$$\begin{aligned} &A_c=\left[A_1-B_1K_FC \quad A_2-B_2K_FC\right], \\ &B_{c1}=\left[-B_1K_F\tau \quad M_1\right],\ B_{c2}=\left[-B_2K_F\tau \quad M_2\right], \\ &B_c=\left[B_{c1} \quad B_{c2}\right], \\ &\tilde{e}(i,j+1)=\begin{bmatrix} e(i,j+1) \\ e_d(i,j+1)\end{bmatrix},\ \tilde{e}(i+1,j)=\begin{bmatrix} e(i+1,j) \\ e_d(i+1,j)\end{bmatrix}, \\ &x_c(i,j)=\left[x^T(i,j+1) \quad x^T(i+1,j)\right]^T. \end{aligned} \quad (28)$$

Then

$$x(i+1,j+1)=A_cx_c(i,j)+B_c\tilde{e}(i,j) \quad (29)$$

Give the Lyapunov function candidate

$$\begin{aligned} &W(i+1,j+1)=V_c(i+1,j+1)+\phi V_o(i+1,j+1) \\ &=x^T(i+1,j+1)P_cx(i+1,j+1) \\ &+\phi e^T(i+1,j+1)Pe(i+1,j+1). \end{aligned} \quad (30)$$

Where P and P_c are positive-definite matrices, and ϕ is a positive scalar. V_o is related with the estimation error that has been discussed in **Theorem 1** and V_c is related with the states of 2-D system. Therefore,

$$\Delta W=\Delta V_c+\phi\Delta V_o \quad (31)$$

One can get as follows:

$$\begin{aligned} \Delta V_o &= V_o(i+1,j+1)-\alpha V_o(i,j+1)-\beta V_o(i+1,j) \\ &\le -\theta_1\left\|\tilde{e}(i,j)\right\|^2 \end{aligned} \quad (32)$$

where θ_1 is a positive scalar.

ΔV_c can be expressed by the formulation as follows:

$$\begin{aligned} \Delta V_c &= \left\{\left[A_cx_c(i,j)+B_c\tilde{e}(i,j)\right]^T P_c\left[A_cx_c(i,j)+B_c\tilde{e}(i,j)\right]\right\} \\ &-\alpha_cV_c(i,j+1)-\beta_cV_c(i+1,j) \\ &=\left[A_cx_c(i,j)\right]^TP_c\left[A_cx_c(i,j)\right]-\alpha_cV_c(i,j+1)-\beta_cV_c(i+1,j) \\ &+\left[B_c\tilde{e}(i,j)\right]^TP_c\left[B_c\tilde{e}(i,j)\right]+2\left[A_cx_c(i,j)\right]^TP_c\left[B_c\tilde{e}(i,j)\right], \end{aligned} \quad (33)$$

where $\alpha_c,\beta_c>0$ and $\alpha_c+\beta_c=1$. Because it's assumed that controller can stabilize the 2-D system in the absence of actuator/sensor fault, there must be a positive scalar υ_1 such that

$$\begin{aligned} &\left[A_cx_c(i,j)\right]^TP_c\left[A_cx_c(i,j)\right]-\alpha_cV_c(i,j+1)-\beta_cV_c(i+1,j) \\ &\le -\upsilon_1\left\|x_c(i,j)\right\|^2 \end{aligned} \quad (34)$$

holds..

Moreover,

$$\begin{aligned} &\left[B_c\tilde{e}(i,j)\right]^TP_c\left[B_c\tilde{e}(i,j)\right]+2\left[A_cx_c(i,j)\right]^TP_c\left[B_c\tilde{e}(i,j)\right] \\ &=\tilde{e}^T(i,j)B_c^TP_cB_c\tilde{e}(i,j)+2x_c^T(i,j)A_c^TP_cB_c\tilde{e}(i,j) \\ &\le\tilde{e}^T(i,j)B_c^TP_cB_c\tilde{e}(i,j)+x_c^T(i,j)x_c(i,j) \\ &+\tilde{e}^T(i,j)B_c^TP_cA_c^TA_cP_cB_c\tilde{e}(i,j) \\ &\le x_c^T(i,j)x_c(i,j)+\tilde{e}^T(i,j)(B_c^TP_cB_c+B_c^TP_cA_c^TA_cP_cB_c)\tilde{e}(i,j) \end{aligned} \quad (35)$$

There must be a positive scalar υ_2 and a positive a scalar θ_2 such that

$$x_c^T(i,j)x_c(i,j)\le\upsilon_2\left\|x_c(i,j)\right\|^2, \quad (36)$$

$$\tilde{e}^T(i,j)(B_c^TP_cB_c+B_c^TP_cA_c^TA_cP_cB_c)\tilde{e}(i,j)\le\theta_2\left\|\tilde{e}(i,j)\right\|^2 \quad (37)$$

holds. Substituting appropriate observer gain matrices such that

$$\Delta W\le-\upsilon_1\left\|x_c(i,j)\right\|^2+\upsilon_2\left\|x_c(i,j)\right\|^2+\theta_2\left\|\tilde{e}(i,j)\right\|^2-\phi\theta_1\left\|\tilde{e}(i,j)\right\|^2 \quad (38)$$

Selecting appropriate observer gain matrices, controller gain, and ϕ such that

$$\begin{aligned} &\upsilon_1-\upsilon_2>0, \\ &\phi\theta_1-\theta_2>0, \\ &\phi>\frac{\theta_2}{\theta_1}. \end{aligned} \quad (39)$$

holds. So, one can obtain that,

$$\Delta W \le -(\upsilon_1 - \upsilon_2)\|x_c(i,j)\|^2 - (\phi\theta_1 - \theta_2)\|\tilde{e}(i,j)\|^2 < 0 \qquad (40)$$

Inequality (39) illustrates that estimation error $\tilde{e}(i,j)$ and system state $x_c(i,j)$ are both asymptotically converge to 0 when $i,j \to \infty$. Hence the closed-loop 2-D system can maintain stable whether or not the sensor/actuator faults occur.

5. Simulation examples

In this part, a numerical example is provided to demonstrate the effectiveness of sensor and actuator faults reconstruction and compensation.

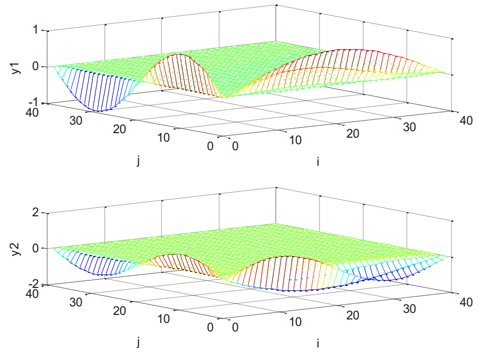

Fig. 1: Output surfaces without sensor/actuator faults: (a) the first output; (b) the second output

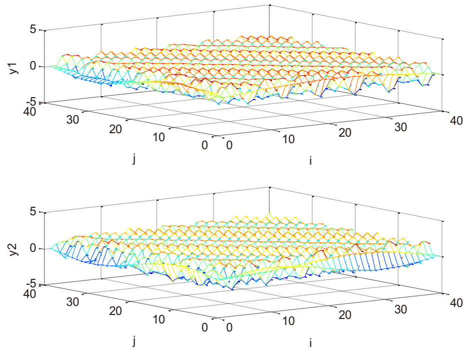

Fig. 2: Output surface with sensor/actuator faults: (a) the first output; (b) the second output

Consider the 2-D system described by the FM-II model (1), the system coefficient matrices and fault vectors are given as follows:

$$A_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0.1 \\ -1 & 0.2 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \qquad (41)$$
$$f_a(i,j) = 0.1f(i-1,j) + 0.3f(i,j-1),$$
$$f_s(i,j) = 5\sin(i+j).$$

The corresponding augmented system coefficient matrices are as follows:

$$\bar{A}_1 = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} 0 & 0.1 & 0 \\ -1 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \quad (42)$$

Assume the output feedback controller is $K_F = 0.3$.

Solving LMI (10) with $\alpha = \beta = 0.5$, one gets the matrices:

$$P = \begin{bmatrix} 1.3228 & -0.0961 & -0.0764 & 0.1670 \\ -0.0961 & 1.7592 & 1.5466 & -0.2545 \\ -0.0764 & 1.5466 & 1.5921 & 0.0518 \\ 0.1670 & -0.2545 & 0.0518 & 4.9641 \end{bmatrix},$$
$$J_1 = \begin{bmatrix} -0.0094 \\ 6.4440 \\ -0.8191 \\ 0.4057 \end{bmatrix}, J_2 = \begin{bmatrix} -0.0344 \\ 6.4833 \\ -0.8233 \\ 0.4071 \end{bmatrix} \qquad (43)$$

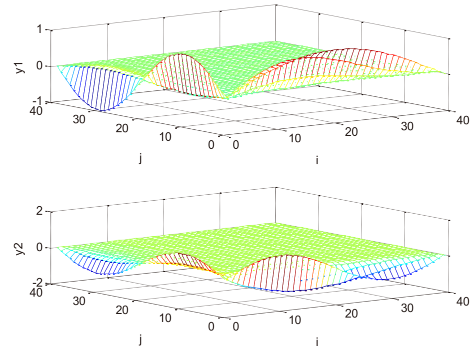

Fig. 3: Fault-compensation control results: (a) the first output; (b) the second output

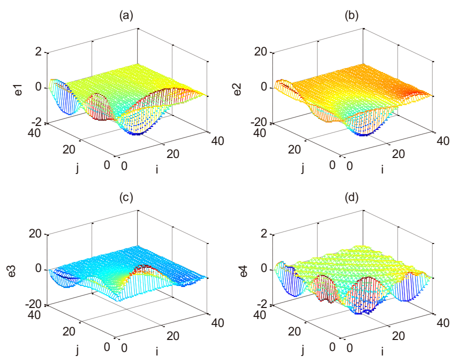

Fig 4: Estimation error $e(i,j)$ of the observer: (a) and (b) show the state estimation errors; (c) and (d) show the fault estimation errors

The simulation results are exhibited in Fig. 1-4. Compared with the original outputs for the fault-free system in Fig. 1, Fig. 2 shows the output of the faulted system without fault-compensation. Fig. 3 shows the compensated outputs for the faulted system and we find that fault compensation scheme is successful in some extent. Fig. 4 shows all estimation errors.

6. Conclusions

In this paper, we use the results of simultaneous estimation of actuator/sensor faults in 2-D linear systems to achieve fault reconstruction and compensation. By designing the asymptotic stable observer, the actuator/sensor faults can be estimated. Hence, the actuator/sensor faults compensation is achieved. Moreover, the considered sensor faults are free of constraints. Finally, simulation shows the excellent fault-tolerant performance.

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