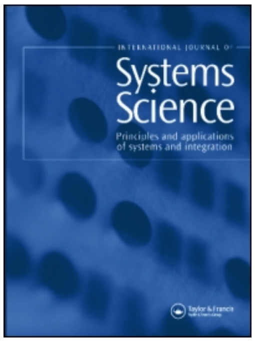

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Reliable H_∞ control for nonlinear discrete-time systems with multiple intermittent faults in sensors or actuators

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ABSTRACT

This study proposes a fault-tolerant control method for stochastic systems with multiple intermittent faults (IFs) and nonlinear disturbances, and both sensor and actuator faults are considered. The occurrence and disappearance of IFs are governed by Markov chain, and its transition probabilities are partly known. Hence, the faulty system can be described by a Markovian jump system (MJS). In order to ensure that the MJS is stochastically stable and satisfies H_∞ performance index, mode-dependent output feedback controllers are modelled using linear matrix inequalities. Numerous sufficient conditions for stochastic stability are obtained on the basis of Lyapunov stability theory. Finally, the effectiveness of the developed method is evaluated on the three-tank system.

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Fault-tolerant control; intermittent faults; Markovian jump system; H_∞ control; linear matrix inequality

1. Introduction

Fault-tolerant control (FTC) schemes for systems subject to faults have received considerable attention owing to the growing requirement for stable and reliable systems (Gao, Ding, & Cecati, 2015; Jiang, Yang, & Shi, 2010; Stoustrup & Blondel, 2004; Wang, Shi, Zhou, & Gao, 2006; Wang, Zhou, & Gao, 2007). The two types of commonly used FTC techniques are active FTC (AFTC) (Dong, Zhong, & Ding, 2012) and passive FTC (PFTC) (Tao, Shen, Fang, & Wang, 2016). In AFTC techniques, the controller is reconfigured on the basis of information from fault detection and diagnosis (He, Wang, Ji, & Zhou, 2010; Zhong, Ding, & Shi, 2009), whereas PFTC techniques depend on a-priori fault information.

The term fault typically refers to a permanent fault (PF) that exists permanently and may deteriorate further if no corrective action is applied after its appearance (Gao, Breikin, & Wang, 2008). Intermittent fault that can recover without any intervention also occur in practical such as network and electronic systems. The occurrence of IFs can lead to poor control performance and even instability (Cui, Dong, Bo, & Juszczyk, 2011; Yang, Jiang and Zhang, 2012). When compared with PFs, the occurrence of IFs poses special problems such as randomness, intermittence and repeatability (Correcher, Garcia, Morant, Quiles, & Rodriguez, 2012). Therefore, a reasonable mathematical description of IFs is the first important step to ensure tolerance of IFs. Considering the features of IFs, stochastic models are increasingly playing a vital role in this field.

Markovian jump systems (MJSs) are typical stochastic systems that have been extensively employed for modelling physical systems with random abrupt variations, and these variations can be depicted by Markov chains (Ma & Boukas, 2009; Seiler & Sengupta, 2005; Wang, Wang, & Wang, 2013; Wang, Liu, & Liu, 2008; Wu, Shi, Su, & Chu, 2013). MJSs have been successfully applied in many areas, including networked control (Seiler & Sengupta, 2005; Wang et al., 2013), filter design (Ma. & Boukas, 2009; Wang. et al., 2008; Wu et al., 2013; Zhang, Zheng, & Xu, 2013) and stability analysis (Goncalves, Fioravanti, & Geromel, 2008; Zhang & Boukas, 2009). However, the application of MJS to describe IFs has not been thoroughly studied. Because the characteristics of IFs satisfy a Markov chain to a large extent, we propose a more practical and precise mathematical description of IFs using a Markov chain. Figure 1 shows IFs described by a Markov chain.

In the studies on MJSs, complete transition probabilities were considered to be available, thus resulting in simplified system analysis and design (Goncalves et al., 2008; Seiler & Sengupta, 2005; Wang et al., 2008). However, in practice, complete knowledge of transition probabilities is difficult or costly to be obtained (Zhang & Boukas, 2009). Therefore, we consider a more practical case with partly known transition probabilities in this study.

FTC of random IFs has seldom been investigated. The present study is an extension of the study of Tao et al. (2016), in which the system was assumed to be linear and the IFs were modelled as a Bernoulli distribution. Nonlinearities are known to occur frequently in practice (Gassara, Hajjaji, Kchaou, & Chaabane, 2014; Shen, Wu, & Park, 2014) and lead to difficulties for control. Therefore, this study proposes a more realistic formulation for a class of nonlinear systems with multiple IFs, where the occurrence of multiple IFs satisfies a Markov chain. The system with multiple IFs in sensors or actuators is converted into an MJS by augmenting its states. In order to ensure that the MJS is stochastically stable and satisfies the guaranteed H_∞ performance index, two mode-dependent output feedback

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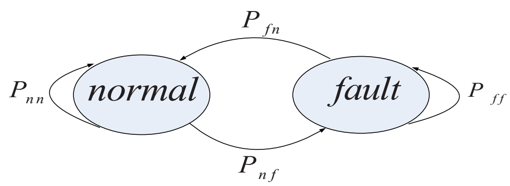

Figure 1. Intermittent faults described by Markov chain.

controllers are modelled on the basis of the linear matrix inequality (LMI) approach.

The rest of this paper is organised as follows. A new mathematical model of multiple IFs and some preliminary knowledge are presented in Section 2. Section 3 shows the main results of the proposed FTC scheme for a nonlinear discrete-time system with multiple sensor and actuator faults. Some simulation results are presented in Section 4 to validate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

Notations: The following notations have been used throughout the paper: $\mathbf{R}^n$ denotes an n-dimensional space, and $\mathbf{R}^{n\times m}$ represents all $n\times m$ real matrices. The superscript T denotes the transpose, and the symbol $*$ denotes the corresponding transposed block in the symmetry block matrix. Further, diag$\{\cdots\}$ represents a block diagonal matrix, and $l_2[0,\infty)$ is the space of square-summable infinite sequence. The norms of sequences $d=\{d(k)\}\in l_2[0,\infty)$ and $z=\{z(k)\}\in l_2[0,\infty)$ can be denoted by $\|d\|_2=\sqrt{\sum_{k=0}^{\infty}|d(k)|^2}$ and $\|z\|_{E_2}=\sqrt{\mathbb{E}[\sum_{k=0}^{\infty}|z(k)|^2]}$, respectively. Further, $E[\bullet]$ indicates the mathematical expectation, and I and $\mathbf{0}$ denote the identity matrix and zero matrix, respectively.

2. Problem formulation and preliminaries

Consider a nonlinear discrete-time system subject to multiple IFs as follows:

$$\begin{cases} x(k+1)=Ax(k)+BM_a(k)u(k)+B_1d(k)+h(x(k)) \\ y(k)=M_s(k)Cx(k)+Dd(k) \\ z(k)=C_1x(k) \end{cases} \tag{1}$$

where $h(x(k))$ is a nonlinear function, and $x(k)\in\mathbf{R}^n$ is the state vector. Further, $u(k)\in\mathbf{R}^m$ is the control input, and $y(k)\in\mathbf{R}^p$ and $z(k)\in\mathbf{R}^r$ are the measured and desired controlled outputs, respectively. Moreover, $A,\ B,\ B_1,\ C,\ D,\ C_1$ are given matrices with appropriate dimensions.

Additionally, $M_s=\text{diag}\{m_{s1},m_{s2},\cdots m_{sp}\}$ and $M_a=\text{diag}\{m_{a1},m_{a2},\cdots m_{am}\}$ are the sensor and actuator fault matrices, respectively. The ith element on the diagonal takes values within $[0,1]$. Further, $m_i=0$ indicates that the ith actuator or sensor is entirely unavailable, while $m_i=1$ indicates that the ith actuator or sensor is available. Moreover, $m_i\in(0,1)$ indicates that the ith actuator or sensor is partly unavailable. Some prior knowledge of the sensor faults is assumed, and a total of N_s possible sensor fault matrices exist. Hence, an N_s-mode Markov chain $\Gamma_s(k)$ is used to describe the IFs occurring in the sensors. The Markov chain takes values in $\Theta=\{1,\ldots,N_s\}$. Further, $p_{ij}=p(\Gamma_s(k+1)=j|\Gamma_s(k)=i)$ denotes the transition probabilities of IFs satisfying $p_{ij}\geq 0,\sum_{j=1}^{N_s}p_{ij}=1$. Similarly, an N_a-mode Markov chain $\Gamma_a(k)$ containing N_a actuator fault matrices is used to describe the IFs occurring in the actuators. Therefore, the Markov chain takes values in $\Psi=\{1,\ldots,N_a\}$ and $p_{ij}=p(\Gamma_a(k+1)=j|\Gamma_a(k)=i)$ satisfying $p_{ij}\geq 0,\sum_{j=1}^{N_a}p_{ij}=1$.

Additionally, $h(x(k))$ is a nonlinear function satisfying the following sector-bounded condition:

$$[h(x(k))-S_1x(k)]^T[h(x(k))-S_2x(k)]\leq 0\ \forall x(k)\in\mathbf{R}^n \tag{2}$$

where real matrices $S_1,S_2\in\mathbf{R}^{n\times n}$ are given, and $S=S_1-S_2$ is a symmetric positive-definite matrix.

The dynamic output feedback controllers are designed with the following form:

$$\begin{cases} x_c(k+1)=A_{c,i}x_c(k)+B_{c,i}y(k) \\ u(k)=C_{c,i}x_c(k) \end{cases} \tag{3}$$

where $x_c(k)\in\mathbf{R}^n$ is the state of the controller, and $A_{c,i}$, $B_{c,i}$ and $C_{c,i}$ are the mode-dependent controller matrices to be designed.

Define

$$\eta(k)=\begin{bmatrix} x(k) \\ x_c(k) \end{bmatrix} \tag{4}$$

By combining (3), (4) and (1), a general form of the closed-loop MJS can be obtained as follows:

$$\begin{cases} \eta(k+1)=\bar{A}_{cl,i}\eta(k)+\bar{B}_{cl,i}d(k)+\bar{E}_{cl,i}h(D\eta(k)) \\ z(k)=\bar{C}_{cl,i}\eta(k) \end{cases} \tag{5}$$

where $\bar{A}_{cl,i}$,$\bar{B}_{cl,i}$ and $\bar{C}_{cl,i}$ denote the matrices of the closed-loop system. The Markov chain takes values in $\bar{\Theta}=\{1,\ldots,\bar{N}\}$ for each $i\in\bar{\Theta}$,$p_{ij}=p(j|i)$, which satisfies $p_{ij}\geq 0,\sum_{j=1}^{\bar{N}}p_{ij}=1$. Let P be the transition probabilities matrix of the MJSs. The initial plant conditions are given by $i(0),x(0)$. Additionally, some elements of the considered transition probabilities are assumed to be available in this study. For instance, for MJS (5) with $\bar{N}$ modes, the transition probabilities matrix can be written as

$$P=\begin{bmatrix} p_{11} & ? & \cdots & p_{1\bar{N}} \\ p_{21} & ? & \cdots & ? \\ \vdots & \vdots & ? & \vdots \\ ? & \cdots & \cdots & p_{\bar{N}\bar{N}} \end{bmatrix} \tag{6}$$

where the question marks (?) represent the unknown elements. Any element could be unknown, but at least one known element exists in each row. That is to say, for a $N\times N$ matrix, the number of unknown elements can be up to N^2-N. For $\forall i\in\bar{\Theta}$, $\bar{\Theta}=\bar{\Theta}_K^i+\bar{\Theta}_{UK}^i$, $\bar{\Theta}_K^i:=\{i:\ p_{ij}\text{ is known}\}$, $\bar{\Theta}_{UK}^i:=\{i:\ p_{ij}\text{ is unknown}\}$.

Remark 2.1: In the proposed formulation, the considered faults are described by using M_s, M_a. For example, suppose that system involves three modes: mode 1 indicates that each sensor is normal, i.e. $M_s(1)=\text{diag}\{1,1\}$; mode 2 indicates that the first and second sensors are partially and absolutely faulty, respectively, i.e. $M_s(2) = \text{diag}\{0.5, 0\}$; mode 3 indicates that the first and second sensors are absolutely and partially faulty, respectively, i.e. $M_s(3) = \text{diag}\{0, 0.5\}$. Furthermore, one knows the following information about the transition probability. Concerning mode 1, the probability of staying in mode 1 is 0.5 with the other transition probabilities unknown. Concerning mode 2, the probabilities from mode 2 to modes 1 and/or 3 are 0.2 and 0.1, respectively, and the other is unknown. Concerning mode 3, the probability from mode 3 to mode 2 is 0.4, and the others are unknown. Then, the transition probability matrix can be written as

$$P = \begin{bmatrix} 0.5 & ? & ? \\ 0.2 & ? & 0.1 \\ ? & 0.4 & ? \end{bmatrix}.$$

The aim of this study is to model mode-dependent dynamic output feedback controllers (3) for system (1) with multiple sensor or actuator faults. Further, the closed-loop system meets the following definitions:

Definition 2.1: (Zhang & Boukas, 2009): System (1) is said to be stochastically stable if for $d(k) \equiv \mathbf{0}$ and every initial condition $i(0) \in \bar{\Theta}$, $x(0) \in R^n$, the following holds:

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{ \|x(k)\|^2 \middle| \, x(0), i(0) \right\} < \infty$$

Definition 2.2: (Mahmoud & Shi, 2002): Given a scalar γ, system (1) is said to be stochastically stable with a H_∞ noise attenuation performance index γ if under zero initial condition, $\|z\|_{E_2} < \gamma \|d\|_2$ holds for all nonzero $d(k) \in l_2[0, \infty)$.

Before proceeding further, let us consider the following lemmas.

Lemma 2.1: The sector-bounded condition (2) is equivalent to

$$\begin{bmatrix} x(k) \\ h(x(k)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & I \end{bmatrix} \begin{bmatrix} x(k) \\ h(x(k)) \end{bmatrix} \le 0.$$

where $R_1 = (S_1^T S_2 + S_2^T S_1)/2$ and $R_2 = -(S_1^T + S_2^T)/2$. Lemma 2.1 can be easily obtained from (2).

Lemma 2.2: (Iwasaki & Hara, 2005): Let us presume that $Z_0(x), Z_1(x), \ldots, Z_l(x)$ are quadratic functions of $x \in R^n$, namely,

$$Z_i(x) = x^T \Phi_i x, \quad i = 0, 1, \ldots, l$$

for $\Phi_i^T = \Phi_i$. If $\tau_1 \ge 0, \tau_2 \ge 0, \ldots, \tau_l \ge 0$ satisfies $\Phi_0 - \sum_{i=1}^{l} \tau_i \Phi_i < 0.$, then $Z_1(x) \le 0, \ldots, Z_l(x) \le 0 \Rightarrow Z_0(x) < 0$ holds.

In the following sections, an FTC method is presented for the nonlinear discrete-time system subject to multiple IFs. It should be noted that sensor and actuator faults are considered in sequence.

Remark 2.2: IFs exist widely in many situations such as net congestion and packet dropout in networked systems (Yang, Jiang, Manivannan, & Singhal, 2005) and electromagnetic interference in electronic systems. Particularly, IFs are the major cause for circuit system failure (Ismaeel & Bhatnagar, 1997). The majority of IFs are activated and inactivated by themselves, and a few of them are caused by noises or disturbances in the environment. Therefore, we can apply the proposed FTC strategy to many practical systems such as aircraft (Yang et al., 2012), mechanical devices, distributed systems (Kandasamy, Hayes, & Murray, 2003) and communication systems.

Remark 2.3: This study is an extension of the work of Tao et al. (2016), which proposes an FTC strategy for discrete-time systems with IFs. In their study, Tao et al. (2016) modelled the IFs as a Bernoulli distribution. However, the considered IFs were additive faults and did not affect the stability of the closed-loop system. In the present study, we investigate multiplicative IFs that can affect system stability, and hence, are more difficult to handle. The previous status significantly affects the occurrence of IFs; however, their model cannot describe the strong correlation between previous status and current fault possibility. In addition, Tao et al. (2016) assumed that all sensor or actuator faults occur simultaneously; however, the occurrence of each fault is independent and stochastic. Therefore, we propose the MJS as a more practical and precise mathematical description.

Remark 2.4: In their study, Tao et al. (2016) assumed the system to be linear. In practical systems, additive nonlinear disturbances occur frequently and often lead to instability or poor performance over time. Therefore, we propose a more reasonable and general fault system that contains sector-bounded nonlinearities.

3. Main results

3.1. Case A: sensor faults

First, let us consider the following system containing only multiple sensor IFs:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_1 d(k) + h(x(k)) \\ y(k) = M_s(k) Cx(k) + Dd(k) \\ z(k) = C_1 x(k) \end{cases} \tag{7}$$

All the parameters in (7) have the same definitions as those in (1). By substituting (3) and (4) into (7), we obtain the following MJS:

$$\begin{cases} \eta(k+1) = A_{cl,i}\eta(k) + B_{cl,i}d(k) + E_{cl,i}h\big(\tilde{D}\eta(k)\big) \\ z(k) = C_{c1,i}\eta(k) \end{cases} \tag{8}$$

where

$$A_{cl,i} = \begin{bmatrix} A & BC_{c,i} \\ B_{c,i}M_s(k)C & A_{c,i} \end{bmatrix} B_{cl,i} = \begin{bmatrix} B_1 \\ B_{c,i}D \end{bmatrix}$$

$$C_{cl,i} = \begin{bmatrix} C_1 & \mathbf{0} \end{bmatrix}, E_{cl,i} = \begin{bmatrix} I \\ \mathbf{0} \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} I & \mathbf{0} \end{bmatrix}$$

According to Section 2, the main task of this part is to design controllers of the format (3), such that system (8) achieves the following two control objectives:

(1) System (8) is stochastically stable
(2) System (8) has a prescribed H_∞ performance index γ_s such that under zero initial condition, $\|z\|_{E_2} < \gamma_s\|d\|_2$ holds for all nonzero $d(k) \in l_2[0, \infty)$.

Theorem 3.1: Given a scalar $\gamma_s > 0$ and considering that the Markov chain transition probabilities matrix is partly available, if there exists a matrix $G_i = {G_i}^T > 0$ and scalar τ such that

$$\begin{bmatrix} -P_K^i G_i - \tau\tilde{D}^T R_1 \tilde{D} & -\tau\tilde{D}^T R_2 & \mathbf{0} & {A_{cl,i}}^T G_K^i & {C_{cl,i}}^T \\ * & -\tau I & \mathbf{0} & {E_{cl,i}}^T G_K^i & \mathbf{0} \\ * & * & -{\gamma_s}^2 I & {B_{cl,i}}^T G_K^i & \mathbf{0} \\ * & * & * & -G_K^i & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} -G_i - \tau\tilde{D}^T R_1 \tilde{D} & -\tau\tilde{D}^T R_2 & \mathbf{0} & {A_{cl,i}}^T G_j & {C_{cl,i}}^T \\ * & -\tau I & \mathbf{0} & {E_{cl,i}}^T G_j & \mathbf{0} \\ * & * & -{\gamma_s}^2 I & {B_{cl,i}}^T G_j & \mathbf{0} \\ * & * & * & -G_j & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0, \forall j \in \Theta_{UK}^i \tag{10}$$

where $P_K^i = \sum_{j\in\Theta_K^i} p_{ij}$ and $G_K^i = \sum_{j\in\Theta_K^i} p_{ij}G_j$, then the MJS (8) is stochastically stable and has a prescribed H_∞ performance index.

Proof: Let us consider the following Lyapunov function:

$$V(\eta(k), i) = \eta^T(k) G_i \eta(k), \forall i \in \Theta$$

By labelling the model at the kth and $k+1$th samples as i and j, respectively, under the condition $d(k) = \mathbf{0}$, we obtain the difference of the Lyapunov function as follows:

$$\begin{aligned} &\mathbb{E}[\Delta V(\eta(k), i)] \\ &= \mathbb{E}[V(\eta(k+1), j|\eta(k), i) - V(\eta(k), i)] \\ &= \eta(k+1)^T \left(\sum_{j\in\Theta_K^i} p_{ij}G_j + \sum_{j\in\Theta_{UK}^i} p_{ij}G_j \right) \eta(k+1) \\ &\quad - \eta(k)^T \left(\sum_{j\in\Theta_K^i} p_{ij}G_i + \sum_{j\in\Theta_{UK}^i} p_{ij}G_i \right) \eta(k) \\ &= \eta(k+1)^T \left(G_K^i + \sum_{j\in\Theta_{UK}^i} p_{ij}G_j \right) \eta(k+1) \\ &\quad - \eta(k)^T \left(P_K^i G_i + \sum_{j\in\Theta_{UK}^i} p_{ij}G_i \right) \eta(k) \\ &= \left(A_{cl,i}\eta(k) + E_{cl,i}h(\tilde{D}\eta(k)) \right)^T \\ &\quad \times \left(G_K^i + \sum_{j\in\Theta_{UK}^i} p_{ij}G_j \right) (A_{cl,i}\eta(k) + E_{cl,i}h(\tilde{D}\eta(k))) \\ &\quad - \eta(k)^T \left(P_K^i G_i + \sum_{j\in\Theta_{UK}^i} p_{ij}G_i \right) \eta(k) = \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} {A_{cl,i}}^T G_K^i A_{cl,i} - P_K^i G_i & {A_{cl,i}}^T G_K^i E_{cl,i} \\ & {E_{cl,i}}^T G_K^i E_{cl,i} \end{bmatrix} \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \\ &\quad + \sum_{j\in\Theta_{UK}^i} p_{ij} \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \begin{bmatrix} {A_{cl,i}}^T G_j A_{cl,i} - G_i & {A_{cl,i}}^T G_j E_{cl,i} \\ * & {E_{cl,i}}^T G_j E_{cl,i} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \end{aligned} \tag{11}$$

By applying the Schur complement to (9) and (10), one obtains

$$\begin{aligned} &\begin{bmatrix} {A_{cl,i}}^T G_K^i A_{cl,i} - G_i + {C_{cl,i}}^T C_{cl,i} & {A_{cl,i}}^T G_K^i E_{cl,i} \\ * & {E_{cl,i}}^T G_K^i E_{cl,i} \end{bmatrix} \\ &\quad - \begin{bmatrix} \tau\tilde{D}^T R_1 \tilde{D} & \tau\tilde{D}^T R_2 \\ * & \tau I \end{bmatrix} < 0 \\ &\begin{bmatrix} {A_{cl,i}}^T G_j A_{cl,i} - G_i + {C_{cl,i}}^T C_{cl,i} & {A_{cl,i}}^T G_j E_{cl,i} \\ * & {E_{cl,i}}^T G_j E_{cl,i} \end{bmatrix} \\ &\quad - \begin{bmatrix} \tau\tilde{D}^T R_1 \tilde{D} & \tau\tilde{D}^T R_2 \\ * & \tau I \end{bmatrix} < 0, \ j \in \Theta_{UK}^i \end{aligned} \tag{12}$$

By applying the congruence transformation $[\,\eta(k)^T \ h(\tilde{D}\eta(k))^T\,]^T$ to (12), we obtain the following inequality:

$$\begin{aligned} &\begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \left\{ \begin{bmatrix} {A_{cl,i}}^T G_K^i A_{cl,i} - G_i + {C_{cl,i}}^T C_{cl,i} & {A_{cl,i}}^T G_K^i E_{cl,i} \\ * & {E_{cl,i}}^T G_K^i E_{cl,i} \end{bmatrix} \right. \\ &\quad \left. - \begin{bmatrix} \tau\tilde{D}^T R_1 \tilde{D} & \tau\tilde{D}^T R_2 \\ * & \tau I \end{bmatrix} \right\} \\ &\begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} < 0 \\ &\begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \left\{ \begin{bmatrix} {A_{cl,i}}^T G_j A_{cl,i} - G_i + {C_{cl,i}}^T C_{cl,i} & {A_{cl,i}}^T G_j E_{cl,i} \\ * & {E_{cl,i}}^T G_j E_{cl,i} \end{bmatrix} \right. \\ &\quad \left. - \begin{bmatrix} \tau\tilde{D}^T R_1 \tilde{D} & \tau\tilde{D}^T R_2 \\ * & \tau I \end{bmatrix} \right\} \\ &\begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} < 0, \ j \in \Theta_{UK}^i \end{aligned}$$

It should be noted that

$$\begin{aligned} &\begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \begin{bmatrix} \tau\tilde{D}^T R_1 \tilde{D} & \tau\tilde{D}^T R_2 \\ * & \tau I \end{bmatrix} \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \\ &= \tau \begin{bmatrix} \tilde{D}\eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & I \end{bmatrix} \begin{bmatrix} \tilde{D}\eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \end{aligned}$$

From Lemmas 2.1 and 2.2, we obtain

$$\begin{bmatrix} {A_{cl,i}}^T G_K^i A_{cl,i} - P_K^i G_i & {A_{cl,i}}^T G_K^i E_{cl,i} \\ * & {E_{cl,i}}^T G_K^i E_{cl,i} \end{bmatrix} < 0$$
$$\begin{bmatrix} {A_{cl,i}}^T G_j A_{cl,i} - G_i & {A_{cl,i}}^T G_j E_{cl,i} \\ * & {E_{cl,i}}^T G_j E_{cl,i} \end{bmatrix} < 0$$

From (11), we have

$$\begin{aligned} \mathbb{E}[\Delta V] \le & -\lambda_{\min} \begin{bmatrix} {A_{cl,i}}^T G_K^i A_{cl,i} - P_K^i G_i & {A_{cl,i}}^T G_K^i E_{cl,i} \\ * & {E_{cl,i}}^T G_K^i E_{cl,i} \end{bmatrix} \\ & \times \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} - \\ & -\lambda_{\min} \begin{bmatrix} {A_{cl,i}}^T G_j A_{cl,i} - G_i & {A_{cl,i}}^T G_j E_{cl,i} \\ * & {E_{cl,i}}^T G_j E_{cl,i} \end{bmatrix} \\ & \times \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \\ \le & -(\beta_1 + \beta_2) \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix}^T \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \\ = & -(\beta_1 + \beta_2) \left\| \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \right\|^2 \\ = & -\beta \left\| \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{bmatrix} \right\|^2 \end{aligned} \tag{13}$$

where $\lambda_{\min}(*)$ indicates the minimum eigenvalue of $*$. Additionally,

$$\begin{aligned} \beta_1 &= \inf \left\{ \lambda_{\min} \left(- \begin{bmatrix} {A_{cl,i}}^T G_K^i A_{cl,i} - P_K^i G_i & {A_{cl,i}}^T G_K^i E_{cl,i} \\ * & {E_{cl,i}}^T G_K^i E_{cl,i} \end{bmatrix} \right) \right\} \\ \beta_2 &= \inf \left\{ \lambda_{\min} \left(- \begin{bmatrix} {A_{cl,i}}^T G_j A_{cl,i} - G_i & {A_{cl,i}}^T G_j E_{cl,i} \\ * & {E_{cl,i}}^T G_j E_{cl,i} \end{bmatrix} \right), \; j \in \Theta_{UK}^i \right\} \\ \beta &= \beta_1 + \beta_2 \end{aligned} \tag{14}$$

From (13) and (14), for any $K > 1$,

$$\begin{aligned} & \mathbb{E}\left\{ \sum_{k=0}^{K} \left\| \begin{matrix} \eta(k) \\ h(\tilde{D}\eta(k)) \end{matrix} \right\|^2 \right\} \\ & \le \frac{1}{\beta} \{ \mathbb{E}[V(\eta(0), 0)] - \mathbb{E}[V(\eta(K+1), K+1)] \} \\ & \le \frac{1}{\beta} \mathbb{E}[V(\eta(0), 0)] \end{aligned}$$

Therefore,

$$\mathbb{E}\left\{ \sum_{k=0}^{K} \|\eta(k)\|^2 \right\} \le \frac{1}{\beta} E[V(\eta(0), 0)] < \infty$$

Hence, the MJS (8) is stochastically stable.

In order to evaluate the H_∞ performance, a new index is introduced as follows:

$$\Xi = \mathbb{E}\left\{ \sum_{k=0}^{\infty} \left[z^T(k) z(k) - {\gamma_s}^2 d^T(k) d(k) \right] \right\}$$

Under the zero initial condition, $V(0) = 0$, $V(\infty) \ge 0$

$$\begin{aligned} \Xi &= \sum_{k=0}^{\infty} \mathbb{E}\left\{ z(k)^T z(k) - \gamma_s^2 d(k)^T d(k) \right\} \\ &= \sum_{k=0}^{\infty} \mathbb{E}\left\{ z(k)^T z(k) - \gamma_s^2 d(k)^T d(k) + \Delta V(k) \right\} \\ &\quad + \mathbb{E}\{V(0)\} - \mathbb{E}\{V(\infty)\} \\ &\le \sum_{k=0}^{\infty} \mathbb{E}\left\{ z(k)^T z(k) - \gamma_s^2 d(k)^T d(k) + \Delta V(k) \right\} \end{aligned}$$

Hence, we obtain

$$\begin{aligned} & \mathbb{E}\left\{ z(k)^T z(k) - {\gamma_s}^2 d(k)^T d(k) + \Delta V \right\} \\ & = \mathbb{E}\{ \big(A_{cl,i}\eta(k) + E_{cl,i} h(D\eta(k)) \\ & \quad + B_{cl,i} d(k) \big)^T \left(G_K^i + \sum_{j \in \Theta_{UK}^i} p_{ij} G_j \right) \big(A_{cl,i}\eta(k) \\ & \quad + E_{cl,i} h(D\eta(k)) + B_{cl,i} d(k) \big) \\ & \quad - \eta(k)^T \left(P_K^i G_i + \sum_{j \in \Theta_{UK}^i} p_{ij} G_i \right) \eta(k) \\ & \quad + \eta(k)^T {C_{cl,i}}^T C_{cl,i} \eta(k) - {\gamma_s}^2 d(k)^T d(k) \} \\ & = \begin{bmatrix} \eta(k) \\ h\left(\tilde{D}\eta(k)\right) \\ d(k) \end{bmatrix}^T \\ & \begin{bmatrix} {A_{cl,i}}^T \hat{G}_i A_{cl,i} - G_i + {C_{cl,i}}^T C_{cl,i} & {A_{cl,i}}^T \hat{G}_i E_{cl,i} & {A_{cl,i}}^T \hat{G}_i B_{cl,i} \\ * & {E_{cl,i}}^T \hat{G}_i E_{cl,i} & {E_{cl,i}}^T \hat{G}_i B_{cl,i} \\ * & * & {B_{cl,i}}^T \hat{G}_i B_{cl,i} - {\gamma_s}^2 I \end{bmatrix} \\ & \begin{bmatrix} \eta(k) \\ h(\tilde{D}\eta(k)) \\ d(k) \end{bmatrix} \\ & \hat{G}_i = G_K^i + \sum_{j \in \Theta_{UK}^i} p_{ij} G_j \end{aligned} \tag{15}$$

From (9) and (10), we obtain

$$\begin{aligned} \Phi_i = & \begin{bmatrix} -P_K^i G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & 0 & {A_{cl,i}}^T G_K^i & {C_{cl,i}}^T \\ * & -\tau I & 0 & {E_{cl,i}}^T G_K^i & 0 \\ * & * & -{\gamma_s}^2 I & {B_{cl,i}}^T G_K^i & 0 \\ * & * & * & -G_K^i & 0 \\ * & * & * & * & -I \end{bmatrix} \\ & + \sum_{j \in \Theta_{UK}^i} p_{ij} \begin{bmatrix} -G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & 0 & {A_{cl,i}}^T G_j & {C_{cl,i}}^T \\ * & -\tau I & 0 & {E_{cl,i}}^T G_j & 0 \\ * & * & -{\gamma_s}^2 I & {B_{cl,i}}^T G_j & 0 \\ * & * & * & -G_j & 0 \\ * & * & * & * & -I \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} -G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & 0 & A_{cl,i}{}^T \hat{G}_i & C_{cl,i}{}^T \\ * & -\tau I & 0 & E_{cl,i}{}^T \hat{G}_i & 0 \\ * & * & -\gamma_s{}^2 I & B_{cl,i}{}^T \hat{G}_i & 0 \\ * & * & * & -\hat{G}_i & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{16}$$

For $\Phi_i < 0$, on the basis of Schur complement, the condition in (16) is equivalent to

$$\begin{bmatrix} -G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & \mathbf{0} \\ * & -\tau I & \mathbf{0} \\ * & * & -\gamma_s{}^2 I \end{bmatrix} - \begin{bmatrix} A^T \hat{G}_i & C^T \\ E^T \hat{G}_i & \mathbf{0} \\ B^T \hat{G}_i & \mathbf{0} \end{bmatrix} \begin{bmatrix} -\hat{G}_i^{-1} & \mathbf{0} \\ * & -I \end{bmatrix} \begin{bmatrix} A^T \hat{G}_i & C^T \\ E^T \hat{G}_i & \mathbf{0} \\ B^T \hat{G}_i & \mathbf{0} \end{bmatrix}^T$$

$$= \begin{bmatrix} -G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & \mathbf{0} \\ * & -\tau I & \mathbf{0} \\ * & * & -\gamma_s{}^2 I \end{bmatrix} - \begin{bmatrix} A^T & C^T \\ E^T & \mathbf{0} \\ B^T{}_i & \mathbf{0} \end{bmatrix} \begin{bmatrix} -\hat{G}_i & \mathbf{0} \\ * & -I \end{bmatrix} \begin{bmatrix} A^T & C^T \\ E^T & \mathbf{0} \\ B^T & \mathbf{0} \end{bmatrix}^T < 0$$

Through some simple matrix operations, we can directly validate that $\Xi < 0$; therefore, $\sum_{k=0}^{\infty} \mathbb{E}\{\|z(k)\|^2\} - \gamma_s{}^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|d(k)\|^2\} < 0$. Hence, the MJS (8) is stochastically stable and has a prescribed performance index γ_s.

Using Schur complement, (16) can be rewritten as

$$\begin{bmatrix} -G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & \mathbf{0} & A_{cl,i}{}^T & C_{cl,i}{}^T \\ * & -\tau I & \mathbf{0} & E_{cl,i}{}^T & \mathbf{0} \\ * & * & -\gamma_s{}^2 I & B_{cl,i}{}^T & \mathbf{0} \\ * & * & * & -\hat{G}_i^{-1} & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{17}$$

This completes the proof.

It should be noted that (17) is a bilinear matrix inequality rather than an LMI. Therefore, we employed the method used in Goncalves et al. (2008) to obtain an equivalent LMI condition and calculate the controller parameters.

Theorem 3.2: For a given scalar $\gamma_s > 0$ and symmetric matrix $Y_i > 0$, if there exist a symmetric matrix X_i, Z_{ij}; real matrices $\hat{M}_i, \hat{L}_i, \hat{F}_i, \hat{H}_i$; and a positive scalar τ such that

$$\begin{bmatrix} \begin{bmatrix} -Y_i - \tau Y_i{}^T R_1 Y_i & -I - \tau Y_i{}^T R_1 & -\tau Y_i{}^T R_2 & \mathbf{0} \\ * & -X_i - \tau R_1 & -\tau R_2 & \mathbf{0} \\ * & * & -\tau I & \mathbf{0} \\ * & * & * & -\gamma_s{}^2 I \end{bmatrix} & \Gamma_i{}^T \\ \Gamma_i & \begin{bmatrix} \Psi_i & \mathbf{0} \\ * & -I \end{bmatrix} \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \hat{Z}_{ij} & \hat{H}_i^T \\ * & Y_j \end{bmatrix} > 0 \tag{19}$$

then the MJS (8) is stochastically stable and has a prescribed H_∞ performance index. Furthermore, the controller matrices can be given as follows:

$$\begin{cases} A_{c,i} = \left(\hat{Y}_I^{-1} - \hat{X}_i\right)^{-1} \left(\hat{M}_i - \hat{X}_i^T A Y_i - \hat{X}_i^T B \hat{L}_i - \hat{F}_i M_s(k) C Y_i\right) Y_i^{-1} \\ B_{c,i} = \left(\hat{Y}_I^{-1} - \hat{X}_i\right)^{-1} \hat{F}_i \\ C_{c,i} = \hat{L}_i (Y_i^{-1})^T \end{cases}$$

where

$$\hat{X}_i = \sum_{j \in \Theta_K^i} p_{ij} X_j + \left(1 - P_K^i\right) \sum_{j \in \Theta_{UK}^i} X_j$$

$$\hat{Y}_I = \left(\sum_{j \in \Theta_K^i} p_{ij} Y_j{}^{-1} + \left(1 - P_K^i\right) \sum_{j \in \Theta_{UK}^i} Y_j{}^{-1} \right)^{-1}$$

$$\Gamma_i{}^T = \begin{bmatrix} Y_i A^T + \hat{L}_i^T B^T & \hat{M}_i^T & Y_i{}^T C_1{}^T \\ A^T & A^T \hat{X}_i + C^T M_s(k)^T \hat{F}_i^T & C_1{}^T \\ I & \hat{X}_i & \mathbf{0} \\ B_1{}^T & B_1{}^T \hat{X}_i + D^T \hat{F}_i^T & \mathbf{0} \end{bmatrix}$$

$$\Psi_i = \begin{bmatrix} -\hat{H}_i - \hat{H}_i^T + \hat{Z}_i & -I \\ * & -\hat{X}_i \end{bmatrix}$$

The definitions of $\hat{M}_i, \hat{L}_i, \hat{F}_i, \hat{H}_i, \hat{Z}_{ij}, \hat{Z}_i$ are given by (27) and (30).

Proof: In accordance with the approach of Goncalves et al. (2008), $G_i \in \mathbf{R}^{2n \times 2n}$ for G_i, G_i^{-1}, T_i can be partitioned as follows:

$$G_i = \begin{bmatrix} X_i & U_i \\ U_i{}^T & \bar{X}_i \end{bmatrix}, G_i^{-1} = \begin{bmatrix} Y_i & V_i \\ V_i{}^T & \bar{Y}_i \end{bmatrix}, T_i = \begin{bmatrix} Y_i & I \\ V_i{}^T & \mathbf{0} \end{bmatrix} \tag{20}$$

where all the blocks are $n \times n$ real symmetric matrices. For $U_i = -\bar{X}_i = Y_i{}^{-1} - X_i$ we can verify that $V_i = Y_i$ and

$$T_i{}^T G_i T_i = \begin{bmatrix} Y_i & I \\ I & X_i \end{bmatrix} \tag{21}$$

Using the partition from (20), we obtain

$$\hat{G}_i = \sum_{j=1}^{N} p_{ij} G_j = \begin{bmatrix} \hat{X}_i & \hat{U}_i \\ \hat{U}_i^T & \hat{\bar{X}}_i \end{bmatrix},$$
$$\hat{G}_i^{-1} = \begin{bmatrix} \hat{R}_1^i & \hat{R}_2^i \\ \left(\hat{R}_2^i\right)^T & \hat{R}_3^i \end{bmatrix}, \hat{Q}_i = \begin{bmatrix} I & \hat{X}_i \\ \mathbf{0} & \hat{U}_i^T \end{bmatrix} \tag{22}$$

Given $\hat{U}_i = -\hat{\bar{X}}_i = \hat{Y}_I^{-1} - \hat{X}_i$, we can verify that $\hat{R}_1^i = \hat{R}_2^i$ and

$$\hat{Q}_i^T \hat{G}_i^{-1} \hat{Q}_i = \begin{bmatrix} \hat{R}_1^i & I \\ I & \hat{X}_i \end{bmatrix} \tag{23}$$

Because $(\hat{R}_1^i)^{-1} = \hat{X}_i - \hat{U}_i(\hat{\bar{X}}_i)^{-1}\hat{U}_i^T$, (23) can be rewritten as

$$\hat{Q}_i^T \hat{G}_i^{-1}\hat{Q}_i = \begin{bmatrix} (\hat{X}_i + \hat{U}_i)^{-1} & I \\ I & \hat{X}_i \end{bmatrix} \tag{24}$$

Using $\hat{U}_i = \hat{Y}_I^{-1} - \hat{X}_i$, the partitioned matrix in (23) can be given as

$$\hat{Q}_i^T \hat{G}_i^{-1}\hat{Q}_i = \begin{bmatrix} \hat{Y}_I & I \\ I & \hat{X}_i \end{bmatrix} \tag{25}$$

Without loss of generality and without assuming that $\hat{U}_i = -\hat{\bar{X}}_i$, we obtain

$$\left(\hat{R}_1^i\right)^{-1} = \hat{X}_i - \hat{U}_i\left(\hat{\bar{X}}_i\right)^{-1}\hat{U}_i^T \geq \sum_{j\in\Theta_K^i} p_{ij}\left(X_j - U_j\bar{X}_i^{-1}U_j^T\right) + \left(1-P_K^i\right)\sum_{j\in\Theta_{UK}^i}\left(X_j - U_j\bar{X}_i^{-1}U_j^T\right) = \sum_{j\in\Theta_K^i} p_{ij}Y_j^{-1} + \left(1-P_K^i\right)\sum_{j\in\Theta_{UK}^i} Y_j^{-1} = \left(\hat{Y}_I\right)^{-1} \tag{26}$$

Therefore,

$$\hat{H}_i = \hat{Y}_I,\ \hat{Z}_{ij} = \hat{Y}_I Y_j^{-1}\hat{Y}_I + \varsigma I\ (\varsigma > 0),\ \hat{Z}_i = Z_K^i + \sum_{j\in\Theta_{UK}^i} p_{ij}Z_{ij} \tag{27}$$

Further, (26) can be written as

$$\hat{H}_i + \hat{H}_i^T - \hat{Z}_i = \hat{Y}_I - \varsigma I \geq \hat{R}_1^i - \varsigma I$$

By considering $\varsigma > 0$ to be sufficiently small, we can verify that

$$\hat{H}_i + \hat{H}_i^T - \hat{Z}_i \geq \hat{R}_1^i \tag{28}$$

By substituting $\hat{R}_1^i$ in the fifth row and fifth block for $\hat{H}_i + \hat{H}_i^T - \hat{Z}_i$ and from (20)–(25), inequality (18) can be rewritten as

$$\begin{bmatrix} -T_i^T G_i T_i - \tau T_i^T \tilde{D}^T R_1 \tilde{D} T_i & -\tau T_i^T \tilde{D}^T R_2 & \mathbf{0} & T_i^T A_{cl,i}{}^T \hat{Q}_i & T_i^T C_{cl,i}{}^T \\ * & -\tau I & \mathbf{0} & E_{cl,i}{}^T \hat{Q}_i & \mathbf{0} \\ * & * & -\gamma_s{}^2 I & B_{cl,i}{}^T \hat{Q}_i & \mathbf{0} \\ * & * & * & -\hat{Q}_i^T \hat{G}_i^{-1}\hat{Q}_i & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{29}$$

The block multiplications can be given as follows:

$$T_i^T G_i T_i = \begin{bmatrix} Y_i & I \\ I & X_i \end{bmatrix},\quad \hat{Q}_i^T \hat{G}_i^{-1}\hat{Q}_i = \begin{bmatrix} \hat{R}_1^i & I \\ I & \hat{X}_i \end{bmatrix}$$

$$T_i^T A_{cl,i}^T \hat{Q}_i = \begin{bmatrix} Y_i A^T + \hat{L}_i^T B^T & \hat{M}_i^T \\ A^T & A^T \hat{X}_i + C^T M_s(k)^T \hat{F}_i^T \end{bmatrix}$$

$$T_i^T C_{cl,i}^T = \begin{bmatrix} Y_i C_1{}^T \\ C_1{}^T \end{bmatrix}$$

$$B_{cl,i}{}^T \hat{Q}_i = \begin{bmatrix} B_1{}^T & B_1{}^T \hat{X}_i + D^T \hat{F}_i^T \end{bmatrix} \tag{30}$$

where

$$U_i = Y_i^{-1} - X_i,\ V_i = Y_i,\ \hat{L}_i = C_{c,i}V_i^T,\ \hat{F}_i = \hat{U}_i B_{c,i}$$
$$\hat{M}_i = \hat{X}_i^T A Y_i + \hat{X}_i^T B \hat{L}_i + \hat{F}_i M_s(k) C Y_i + \hat{U}_i A_{c,i} V_i$$

The controller parameters can then be derived as follows:

$$\begin{cases} A_{c,i} = \left(\hat{Y}_I^{-1} - \hat{X}_i\right)^{-1}\left(\hat{M}_i - \hat{X}_i^T A Y_i - \hat{X}_i^T B \hat{L}_i - \hat{F}_i M_s(k) C Y_i\right) Y_i^{-1} \\ B_{c,i} = \left(\hat{Y}_I^{-1} - \hat{X}_i\right)^{-1}\hat{F}_i \\ C_{c,i} = \hat{L}_i\left(Y_i^{-1}\right)^T \end{cases} \tag{31}$$

By applying the congruence transformation $\mathrm{diag}\{T_i^{-T}, I, I, \hat{Q}_i^{-T}, I\}$ to (29), we obtain the following inequality:

$$\begin{bmatrix} -G_i - \tau\tilde{D}^T R_1 \tilde{D} & -\tau\tilde{D}^T R_2 & \mathbf{0} & A_{cl,i}{}^T & C_{cl,i}{}^T \\ * & -\tau I & \mathbf{0} & E_{cl,i}{}^T & \mathbf{0} \\ * & * & -\gamma_s{}^2 I & B_{cl,i}{}^T & \mathbf{0} \\ * & * & * & -\hat{G}_i^{-1} & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{32}$$

This completes the proof.

3.2. Case B: actuator faults

Consider a class of systems with only actuator IFs as follows:

$$\begin{cases} x(k+1) = Ax(k) + BM_a(k)u(k) + B_1 d(k) + h(x(k)) \\ y(k) = Cx(k) + Dd(k) \\ z(k) = C_1 x(k) \end{cases} \tag{33}$$

where all the parameters have the same definitions as those in (1). By substituting (3) and (4) into (33), we obtain the following MJS:

$$\begin{cases} \eta(k+1) = A_{cl,i}\eta(k) + B_{cl,i}d(k) + E_{cl,i}h(\tilde{D}\eta(k)) \\ z(k) = C_{c1,i}\eta(k) \end{cases} \tag{34}$$

where

$$A_{cl,i} = \begin{bmatrix} A & BM_a(k)C_{c,i} \\ B_{c,i}C & A_{c,i} \end{bmatrix},\ B_{cl,i} = \begin{bmatrix} B_1 \\ B_{c,i}D \end{bmatrix}$$
$$C_{cl,i} = \begin{bmatrix} C_1 & \mathbf{0} \end{bmatrix},\ E_{cl,i} = \begin{bmatrix} I \\ \mathbf{0} \end{bmatrix},\ \tilde{D} = \begin{bmatrix} I & \mathbf{0} \end{bmatrix}$$

According to Section 2, the main task of this part is to design controllers with the form (3) such that the MJS (34) achieves the following two control objectives:

(1) System (34) is stochastically stable.
(2) System (34) has a prescribed H_∞ performance index γ_a, namely, under zero initial condition, $\|z\|_{E_2} < \gamma_a\|d\|_2$ for all nonzero $d(k) \in l_2[0,\infty)$.

Theorem 3.3: For a prescribed scalar $\gamma_a > 0$ and considering that the Markov chain's transition probabilities matrix is partly

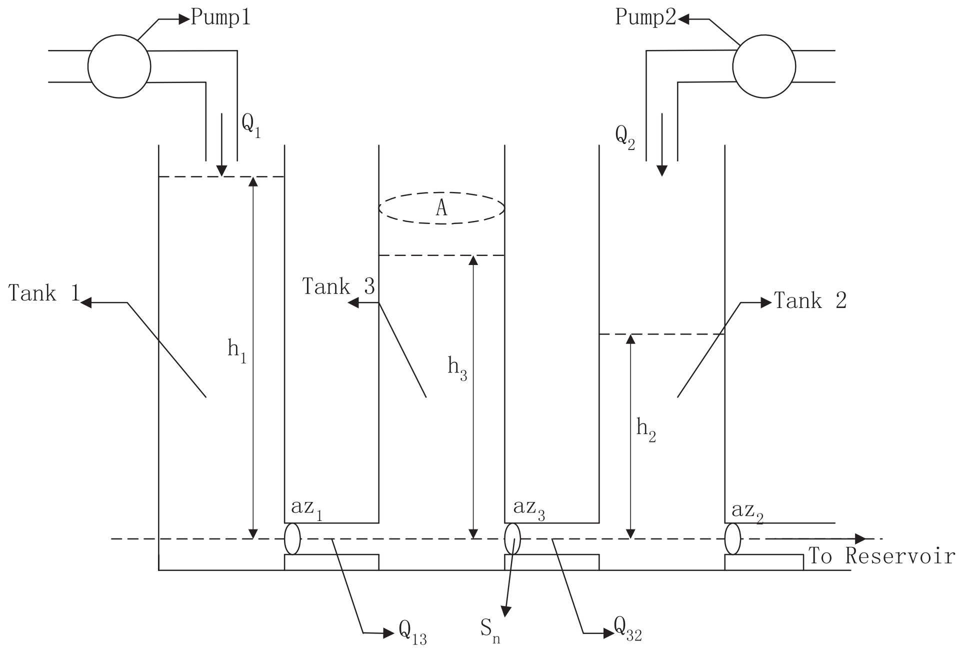

Figure 2. Three-tank system DTS200 (Xie et al., 1999).

known, if there exist a matrix $G_i = G_i^T > 0$ and a scalar τ, we obtain

$$\begin{bmatrix} -P_K^i G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & \mathbf{0} & A_{cl,i}{}^T G_K^i & C_{cl,i}{}^T \\ * & -\tau I & \mathbf{0} & E_{cl,i}{}^T G_K^i & \mathbf{0} \\ * & * & -\gamma_a{}^2 I & B_{cl,i}{}^T G_K^i & \mathbf{0} \\ * & * & * & -G_K^i & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{35}$$

$$\begin{bmatrix} -G_i - \tau \tilde{D}^T R_1 \tilde{D} & -\tau \tilde{D}^T R_2 & \mathbf{0} & A_{cl,i}{}^T G_j & C_{cl,i}{}^T \\ * & -\tau I & \mathbf{0} & E_{cl,i}{}^T G_j & \mathbf{0} \\ * & * & -\gamma_a{}^2 I & B_{cl,i}{}^T G_j & \mathbf{0} \\ * & * & * & -G_j & \mathbf{0} \\ * & * & * & * & -I \end{bmatrix} < 0, \forall j \in \Psi_{UK}^i \tag{36}$$

where $P_K^i = \sum_{j \in \Psi_K^i} p_{ij}$, $G_K^i = \sum_{j \in \Psi_K^i} p_{ij} G_j$, then the closed-loop MJS (34) is stochastically stable and has a prescribed H_∞ performance index.

The proof of Theorem 3.3 is similar to that of Theorem 3.1 and has been omitted for simplicity.

Theorem 3.4: For a given scalar $\gamma_a > 0$ and symmetric matrix $Y_i > 0$, if there exist a symmetric matrix X_i, Z_{ij}; real matrices M_i, L_i, F_i, H_i; and a positive scalar τ such that

$$\begin{bmatrix} \begin{bmatrix} -Y_i - \tau Y_i{}^T R_1 Y_i & -I - \tau Y_i{}^T R_1 & -\tau Y_i{}^T R_2 & \mathbf{0} \\ * & -X_i - \tau R_1 & -\tau R_2 & \mathbf{0} \\ * & * & -\tau I & \mathbf{0} \\ * & * & * & -\gamma_a{}^2 I \end{bmatrix} & \Gamma_i{}^T \\ \Gamma_i & \begin{bmatrix} -\hat{H}_i - \hat{H}_i^T + \hat{Z}_i & -I & \mathbf{0} \\ * & -\hat{X}_i & \mathbf{0} \\ * & * & -I \end{bmatrix} \end{bmatrix} < 0 \tag{37}$$

$$\begin{bmatrix} \hat{Z}_{ij} & \hat{H}_i^T \\ * & Y_j \end{bmatrix} > 0, \tag{38}$$

then the MJS (34) is stochastically stable and has a H_∞ performance index. Furthermore, the controller parameters can be determined as follows:

$$\begin{cases} A_{c,i} = \left(\hat{Y}_I^{-1} - \hat{X}_i\right)^{-1} \left(\hat{M}_i - \hat{X}_i A Y_i - \hat{X}_i B M_a(k) \hat{L}_i - \hat{F}_i C Y_i\right) Y_i^{-1} \\ B_{c,i} = \left(\hat{Y}_I^{-1} - \hat{X}_i\right)^{-1} \hat{F}_i \\ C_{c,i} = \hat{L}_i \left(Y_i{}^{-1}\right)^T \end{cases}$$

where

$$\Gamma_i{}^T = \begin{bmatrix} Y_i A^T + \hat{L}_i^T M_a(k)^T B^T & \hat{M}_i^T & Y_i{}^T C_1{}^T \\ A^T & A^T \hat{X}_i + C^T \hat{F}_i^T & C_1{}^T \\ I & \hat{X}_i & \mathbf{0} \\ B_1{}^T & B_1{}^T \hat{X}_i + D^T \hat{F}_i^T & \mathbf{0} \end{bmatrix}$$

The proof of Theorem 3.4 is similar to that of Theorem 3.2 and has been omitted for simplicity.

4. Illustrative example

Here, a three-tank system called DTS200 is provided to illustrate the effectiveness of the proposed FTC strategy. DTS200 is a nonlinear continuous-time system and all parameters can be found in Xie, Zhou, & Jin (1999). The layout of DTS200 is shown in Figure 2.

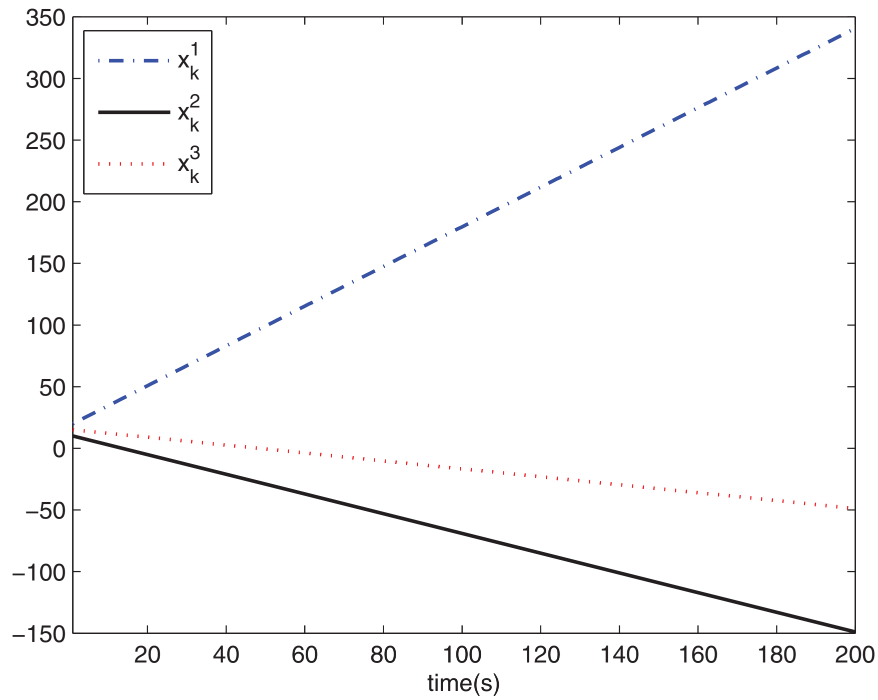

Figure 3. State response of the uncontrolled system.

With a sampling period of 1 s, one can obtain a discrete-time model as follows:

$$\begin{cases} \xi(k+1) = A\xi(k) + B\upsilon(k) + B_1 d(k) + h(\xi(k)) \\ y_\xi(k) = C\xi(k) + Dd(k) \\ \varsigma(k) = C_1\xi(k) \end{cases} \tag{39}$$

where

$$h(\xi) = \frac{1}{S_A}\begin{bmatrix} -Q_{13} \\ Q_{32} - Q_{20} \\ Q_{13} - Q_{32} \end{bmatrix} = \frac{1}{S_A}\begin{bmatrix} -az_1 S_n \operatorname{sgn}(h_1 - h_3)\left(2g\,|h_1 - h_3|\right)^{1/2} \\ az_3 S_n \operatorname{sgn}(h_3 - h_2)\left(2g\,|h_3 - h_2|\right)^{1/2} \\ -az_2 S_n \left(2gh_2\right)^{1/2} \\ az_1 S_n \operatorname{sgn}(h_1 - h_3)\left(2g\,|h_1 - h_3|\right)^{1/2} - \\ az_3 S_n \operatorname{sgn}(h_3 - h_2)\left(2g\,|h_3 - h_2|\right)^{1/2} \end{bmatrix}$$

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \hat{=} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, \upsilon \hat{=} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, B = \frac{1}{S_A}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, A = B_1 = D = C = I_3,$$
$$S_A = 154, S_n = 0.5, Q_{1\max} = Q_{2\max} = 100,$$
$$az_1 = 0.5, az_2 = az_3 = 0.6, g = 981.$$

Given the set point for the desired controlled output as $\varsigma_r = [\,30\ 20\,]^T$, one gets the steady liquid levels $x_s = [\,30\ 20\ 24.0984\,]^T$ and input $u_s = [\,26.9014\ 32.5259\,]^T$. Define $x = \xi - x_s$, $u = \upsilon - u_s$, $z_d = \varsigma - \varsigma_r$, $y = y_\xi - y_r$, the tracking problem can be transformed into a stabilisation problem. The new model can then be derived as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_1 d(k) + g(x(k)) \\ y(k) = Cx(k) + Dd(k) \\ z_d(k) = C_1 x(k) \end{cases}$$

where $g(x(k)) = h(x(k) + x_s(k)) - h(x_s(k))$. It is bounded by

$$S_1 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}, \ S_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A, B, B_1, C, C_1, D have the same definitions as those in (39). The disturbance can then be given as follows:

$$d(k) = \begin{bmatrix} 0.7\exp(-0.1k)\sin(0.01\pi k) \\ 0.3\sin(0.01\pi k) \\ 0.5\exp(-0.1k)\sin(0.01\pi k) \end{bmatrix}$$

The transition probability matrix is given by

$$P = \begin{bmatrix} 0.5 & ? & ? \\ ? & 0.6 & ? \\ ? & ? & 0.7 \end{bmatrix}$$

Let $\gamma_s = \gamma_a = 4$ and

$$Y_1 = \begin{bmatrix} 3 & -1 & 1.5 \\ -1 & 5 & 0 \\ 1.5 & 0 & 2 \end{bmatrix}, \ Y_2 = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}, \ Y_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

Case A: Let us suppose that the system involves three modes, and the mode data are given as follows. Mode 1 indicates that each sensor is normal, and its matrix is $M_s = \operatorname{diag}\{1,\ 1,\ 1\}$.

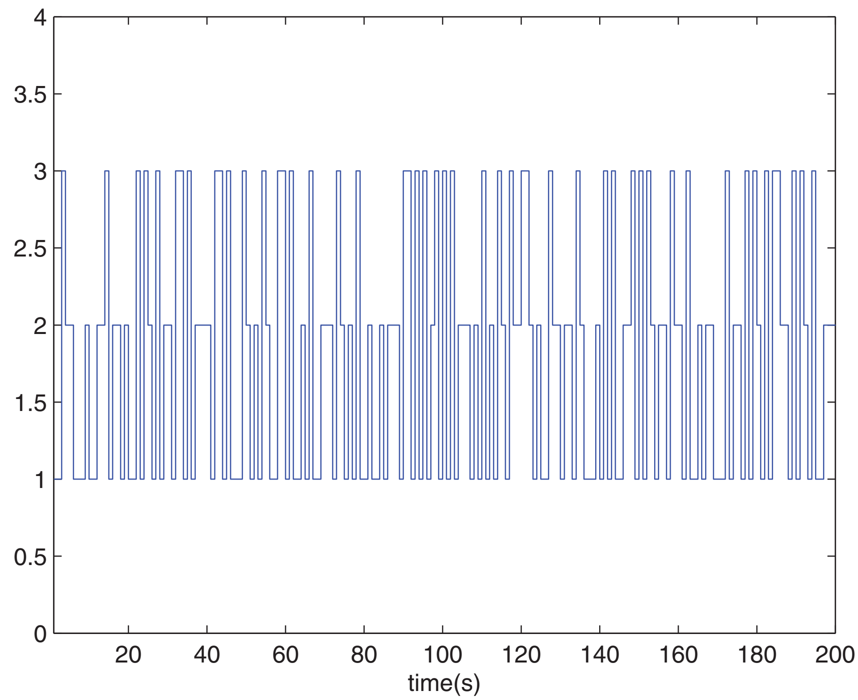

Figure 4. Occurrence of intermittent faults in the controlled system for Case A.

Mode 2 indicates that the first two sensors are partially faulty and the third sensor is normal, i.e. $M_s = \text{diag}\{0.6,\ 0.6,\ 1\}$. Mode 3 indicates that the first sensor is absolutely faulty and the other sensors are partially faulty, i.e. $M_s = \text{diag}\{0,\ 0.6,\ 0.6\}$.

By solving the matrix inequalities (18) and (19), one can obtain the following controllers:

$$A_{c1} = \begin{bmatrix} -0.9551 & 0.0121 & 0.9300 \\ 0.0645 & -0.9990 & -0.0407 \\ 0.0256 & -0.0163 & 0.0247 \end{bmatrix},$$

$$B_{c1} = \begin{bmatrix} 1.1857 & -0.0034 & -0.0092 \\ -0.0005 & 1.0809 & -0.0003 \\ 0.0366 & 0.0153 & 1.0635 \end{bmatrix}$$

$$C_{c1} = \begin{bmatrix} -136.5138 & 5.1504 & 145.7419 \\ 8.3764 & -158.1357 & -1.1160 \end{bmatrix},$$

$$A_{c2} = \begin{bmatrix} -0.9564 & -0.2225 & 0.2004 \\ 0.0298 & -0.9838 & 0.0131 \\ 0.0113 & 0.0086 & -0.0027 \end{bmatrix}$$

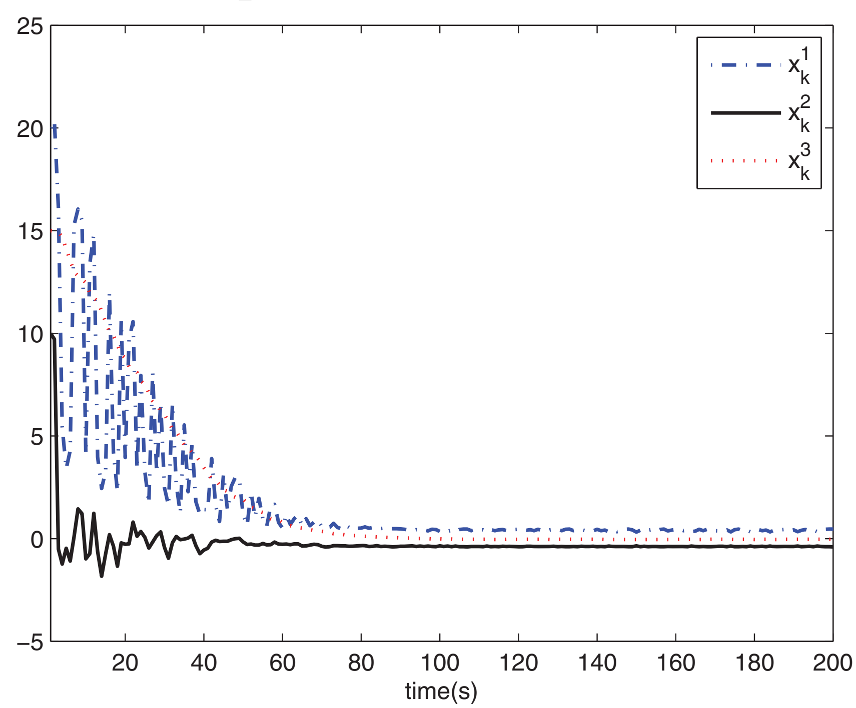

Figure 5. State response of the controlled system for Case A.

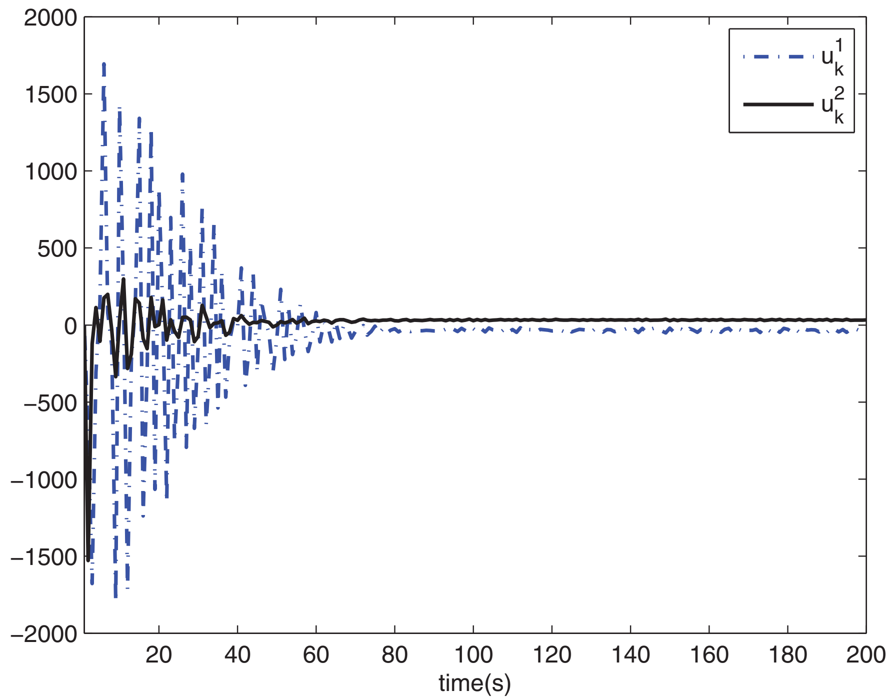

Figure 6. Designed input signal for Case A.

$$B_{c2} = \begin{bmatrix} 1.5239 & -0.0223 & -0.0348 \\ 0.0055 & 1.5324 & -0.0862 \\ 0.0851 & 0.0410 & 1.0741 \end{bmatrix},$$

$$C_{c2} = \begin{bmatrix} -137.0221 & -42.8346 & 34.9264 \\ -0.9884 & -159.1984 & -10.2478 \end{bmatrix}$$

$$A_{c3} = \begin{bmatrix} -0.9286 & -0.0085 & 0.1598 \\ -0.0486 & -0.9759 & -0.0556 \\ 0.0240 & 0.0294 & 0.0087 \end{bmatrix},$$

$$B_{c3} = \begin{bmatrix} 2.0645 & -0.0513 & 0.0199 \\ 0.0624 & 1.8243 & 0.0677 \\ -0.0998 & 0.0181 & 2.2085 \end{bmatrix}$$

$$C_{c3} = \begin{bmatrix} -130.1840 & -10.7829 & 32.4457 \\ 4.1477 & -153.7280 & -6.4121 \end{bmatrix}$$

Case B: Let us suppose that the system involves three modes, and the mode data are given as follows. Mode 1 indicates that

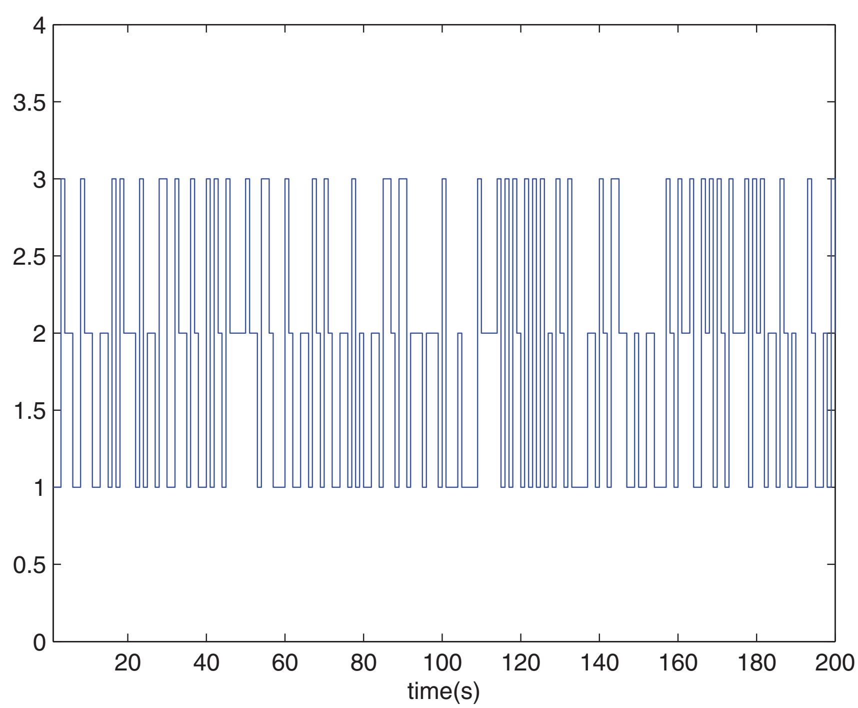

Figure 7. Occurrence of intermittent faults in controlled system for Case B.

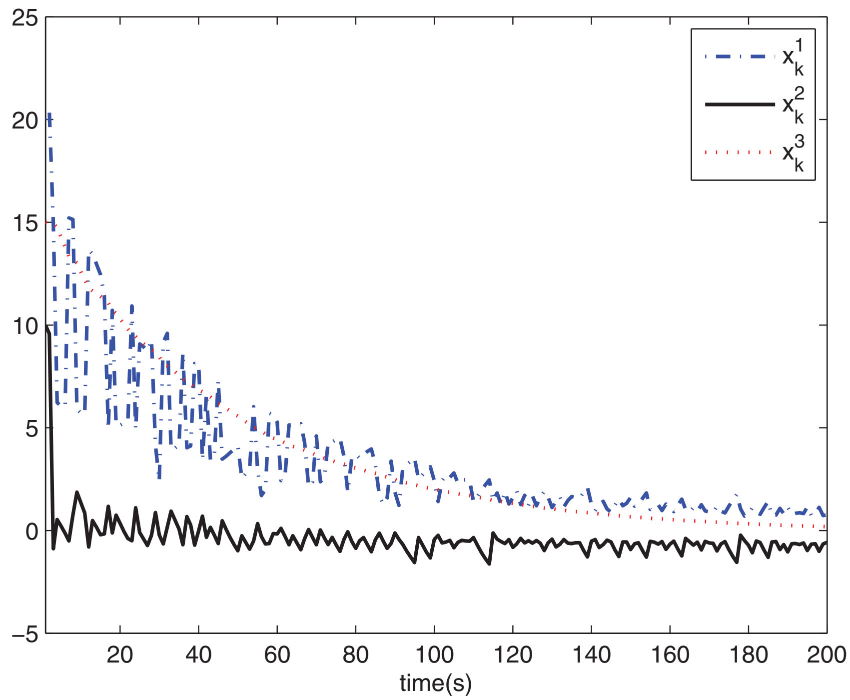

Figure 8. State response of controlled system for Case B.

each actuator is normal, and its matrix is $M_a = \mathrm{diag}\{1,\ 1\}$. Mode 2 indicates that the first and second actuators are partially and absolutely faulty, respectively, i.e. $M_a = \mathrm{diag}\{0.6,\ 0\}$. Mode 3 indicates that the first and second actuators are absolutely and partially faulty, respectively, i.e. $M_a = \mathrm{diag}\{0,\ 0.6\}$.

By solving the matrix inequalities (37) and (38), one can obtain the following controllers:

$$A_{c1} = \begin{bmatrix} -0.9811 & 0.0278 & 0.9299 \\ 0.0683 & -1.0261 & -0.0645 \\ 0.0871 & -0.0895 & -0.0635 \end{bmatrix},$$

$$B_{c1} = \begin{bmatrix} 1.1898 & -0.0277 & -0.0105 \\ -0.0304 & 1.1006 & 0.0078 \\ -0.0785 & 0.0709 & 1.099 \end{bmatrix}$$

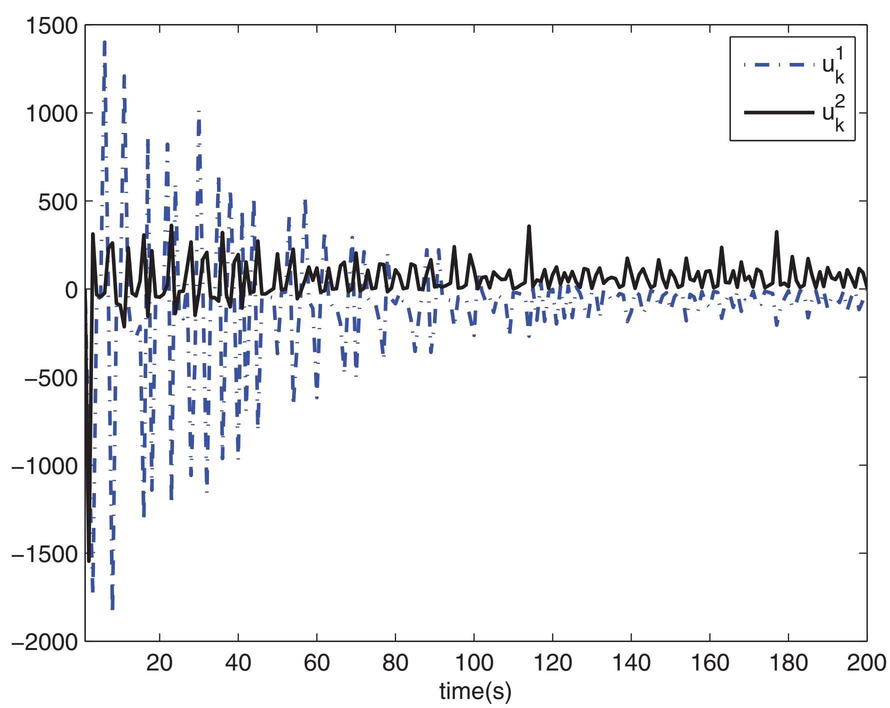

Figure 9. Designed input signal for Case B.

$$C_{c1} = \begin{bmatrix} -138.5521 & 4.6917 & 145.5844 \\ 6.7145 & -158.6575 & -2.1528 \end{bmatrix},$$

$$A_{c2} = \begin{bmatrix} -0.1068 & 0.0633 & -0.0381 \\ 0.0028 & -0.0363 & -0.0015 \\ -0.0163 & 0.0876 & 0.0169 \end{bmatrix}$$

$$B_{c2} = \begin{bmatrix} 1.2009 & -0.0529 & -0.0082 \\ -0.0035 & 1.0948 & -0.0007 \\ -0.0908 & 0.0562 & 1.0936 \end{bmatrix},$$

$$C_{c2} = \begin{bmatrix} -25.3499 & -5.7851 & 7.6842 \\ -0.3135 & -35.0828 & -1.5101 \end{bmatrix}$$

$$A_{c3} = \begin{bmatrix} -0.8168 & -0.0703 & 0.1602 \\ 0.1261 & -1.0468 & -0.0617 \\ 0.0504 & -0.01385 & -0.0104 \end{bmatrix},$$

$$B_{c3} = \begin{bmatrix} 1.1993 & -0.0238 & -0.0122 \\ -0.0027 & 1.1016 & 0.0012 \\ -0.1343 & 0.0821 & 1.1029 \end{bmatrix}$$

$$C_{c3} = \begin{bmatrix} -146.7171 & -22.1762 & 36.7042 \\ 31.2661 & -205.8505 & -13.7798 \end{bmatrix}$$

Figures 3–9 show the simulation results. The state response of the uncontrolled system is shown in Figure 3, and we can see that the system is unstable. Figures 4 and 7 show the occurrence of IFs for cases A and B, respectively. Figures 5 and 8 show the state responses of the controlled systems for cases A and B, respectively. We can see that the designed controllers ensure that the system is stable in the presence of multiple IFs, and the system rapidly converges after the occurrence of multiple sensor or actuator faults. The designed control signals for cases A and B are shown in Figures 6 and 9, respectively.

5. Conclusions

In this study, we proposed a reliable H_∞ control for a class of nonlinear discrete-time systems subject to multiple IFs in sensors or actuators. Considering the features of IFs, the system was transformed into an MJS. In order to ensure that the designed MJS is stochastically stable and has the prescribed H_∞ performance index for all the faults, dynamic output-feedback controllers were modelled using LMIs. The proposed method was verified on the three-tank system through simulation tests.

Disclosure statement

No potential conflict of interest was reported by the authors.

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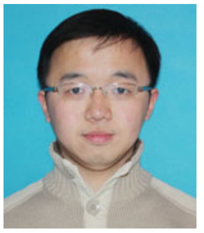

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